Renormalization and UV behaviour of 5D gauge theories

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Outline

- Motivation
- Running of couplings (RGEs): from 4D to 5D
- Renormalization of 5D gauge theories
- Conclusions

Asymptotic Grand Unification Theories (GUTs)

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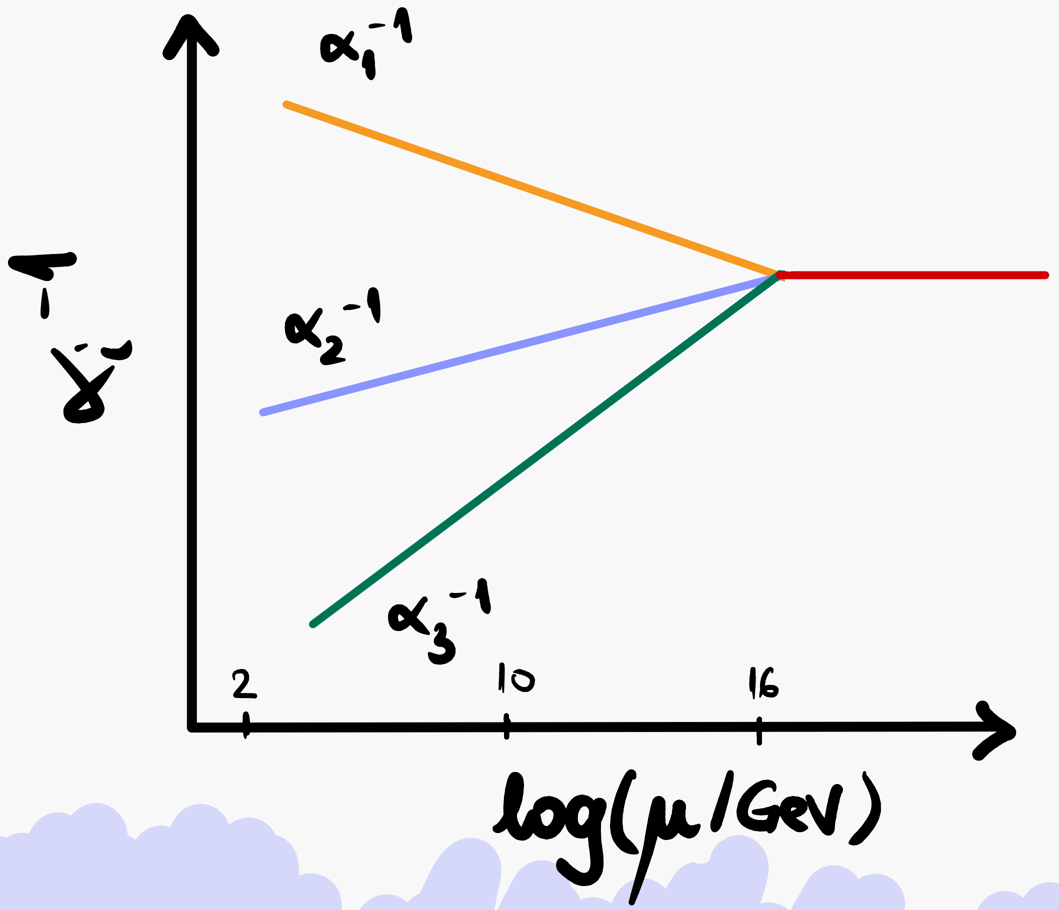
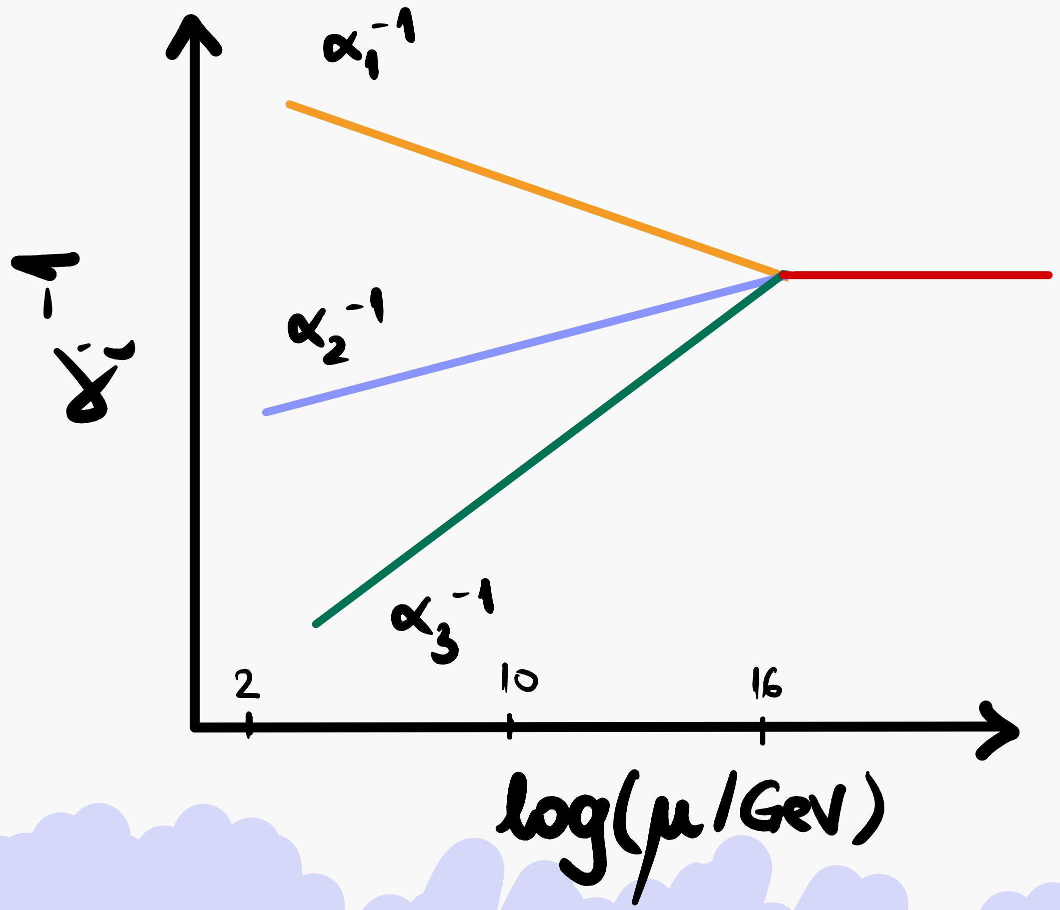


4D theories

standard idea of gauge coupling unification

SU(5), SO(10) ...

Asymptotic Grand Unification Theories (GUTs)



extra dimensional theories (5D)

same models SU(5), SO(10) ...

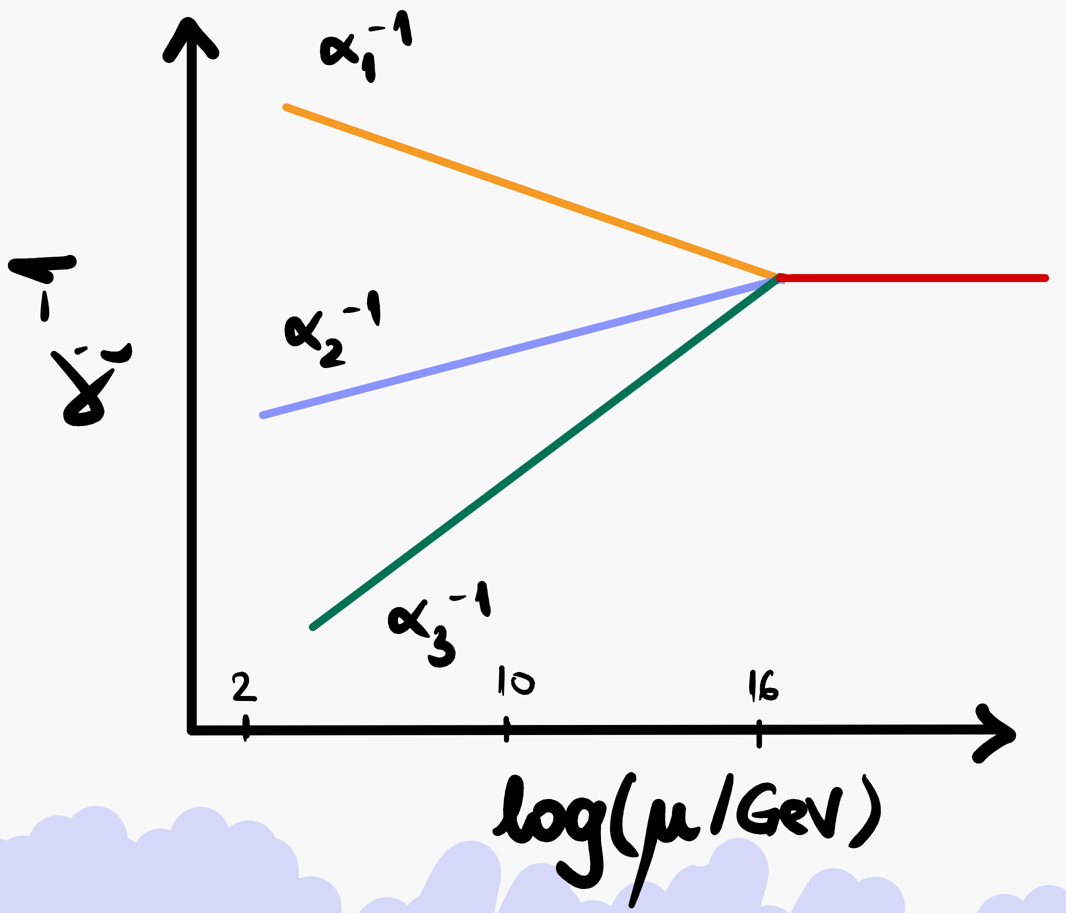
different construction

4D theories

standard idea of gauge coupling unification

SU(5), SO(10) ...

Asymptotic Grand Unification Theories (GUTs)



extra dimensional theories (5D)

same models SU(5), SO(10) ...

different construction

4D theories

standard idea of gauge coupling unification

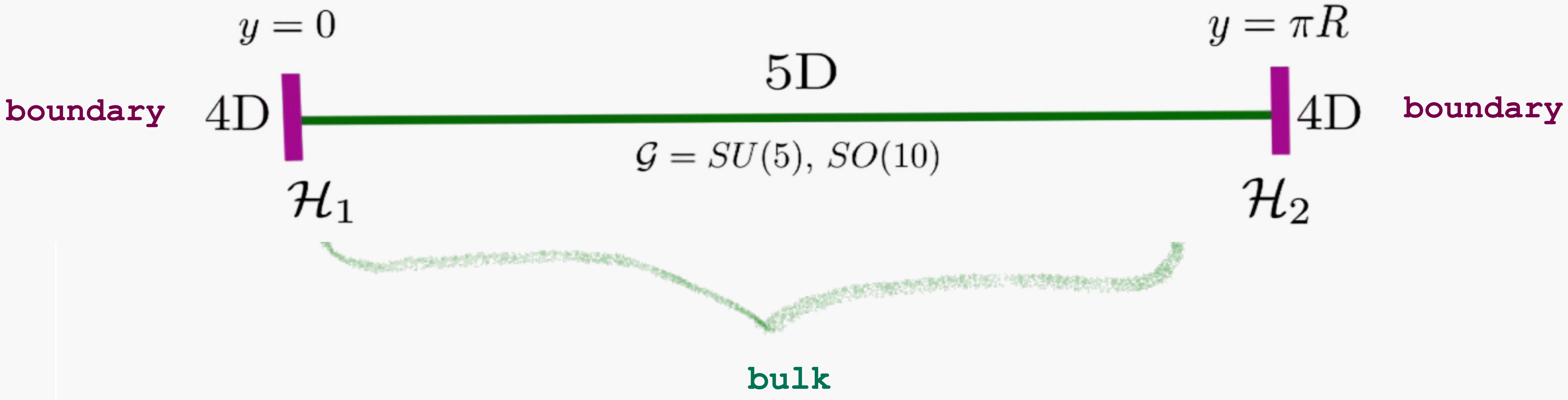
SU(5), SO(10) ...

Why?

- lower unification scale
- less parameters
- hierarchy problem

Asymptotic GUTs

- **5D models**: one extra dimension[1, 2] compactified on $K = S^1/\mathbb{Z}_2 \times \mathbb{Z}'_2$



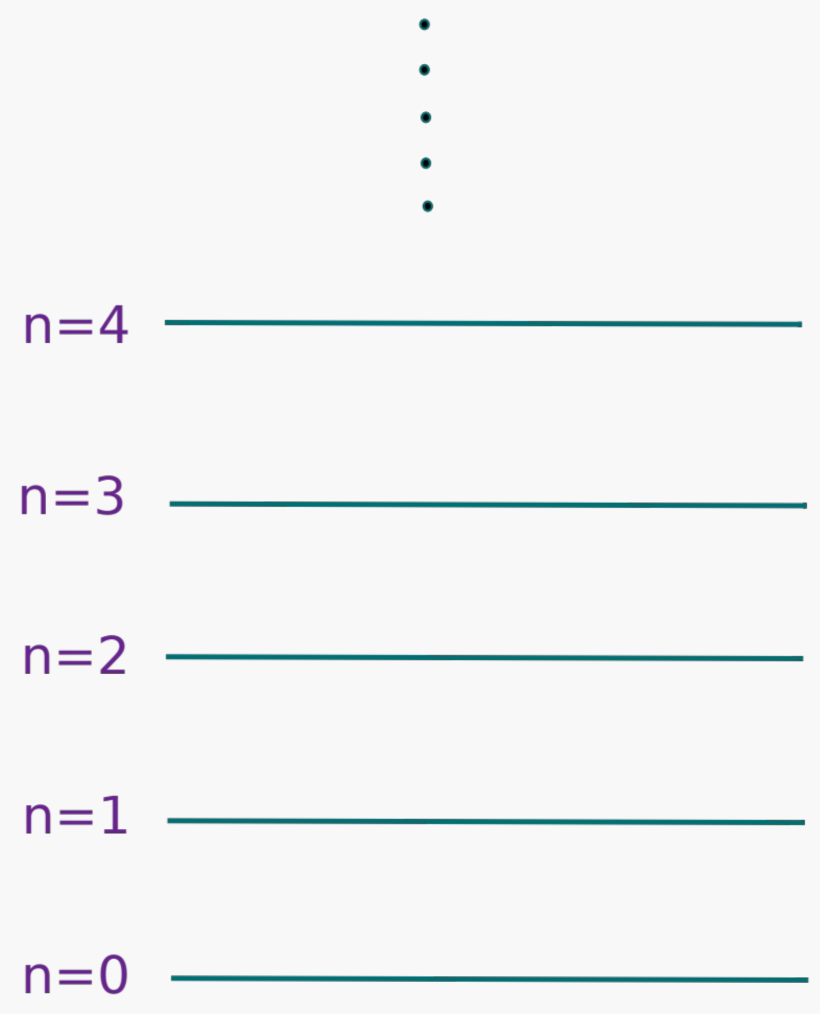
[1] A. Hebecker, J. March-Russell, Nuclear Phys. B 625 (2002)
[2] G. Cacciapaglia, arXiv:2309.10098 (2023)

Any 5D field

$$\Phi_{5\text{D}}$$

(scalar, fermion, gauge boson)

Infinite tower of 4D fields



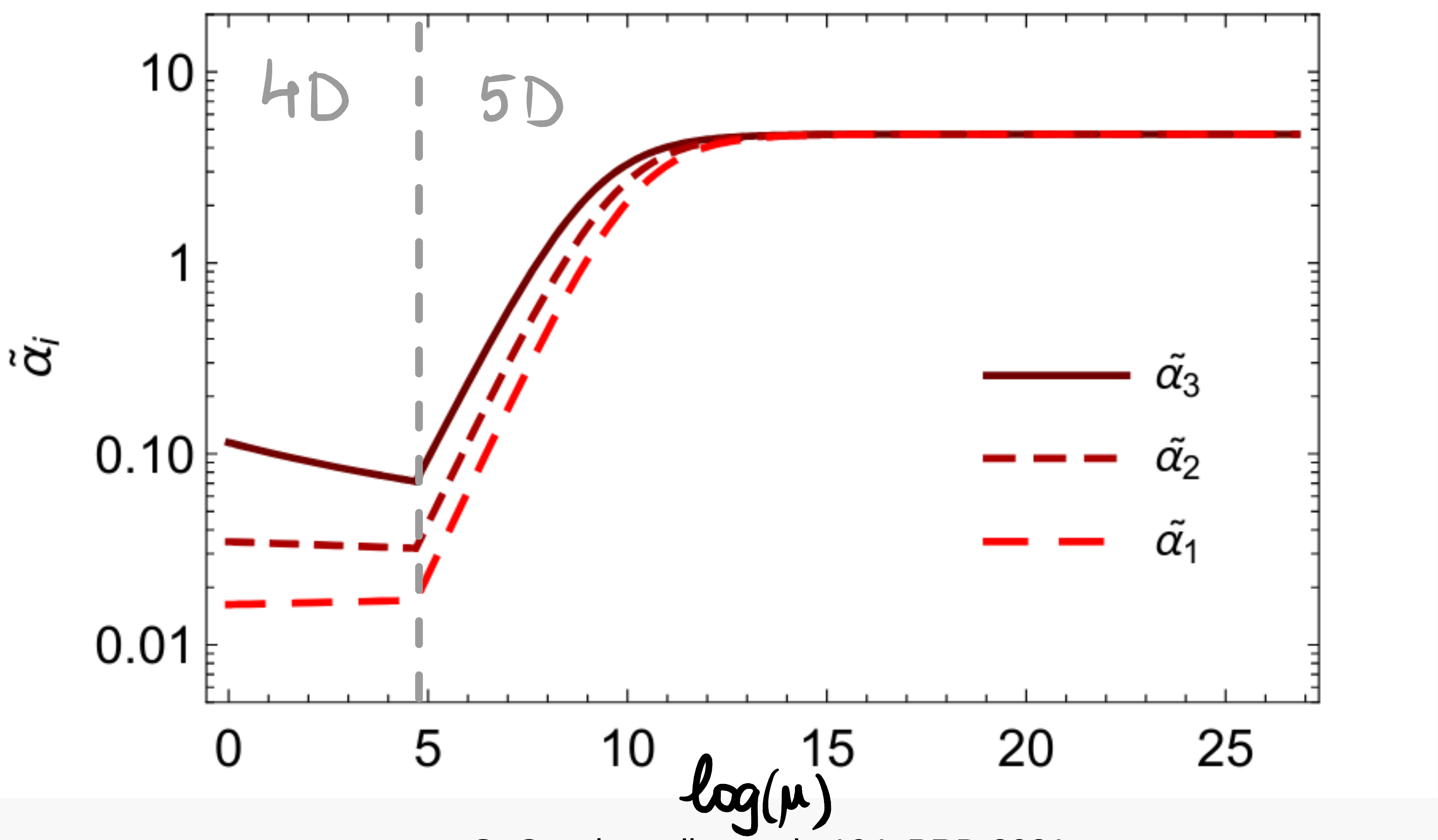
RGEs: from 4D to 5D

- 4D: logarithmic running $$16\pi^2 \frac{dg_i}{dt} = b^i_{SM}\, g_i^3$$

- 5D: power-law running [2]

$$16\pi^2 \frac{dg_i}{dt} = b^i_{SM}\, g_i^3 + (S(t)-1) b^i_5 g_i^3$$

where $S(t) = \begin{cases} \mu\ \mathrm{R}{=}M_Z\ \mathrm{R}\ e^t, & \text{for } \mu \geq 1/R \\ 1, & \text{otherwise} \end{cases}$



G. Cacciapaglia ,et.al., 104, PRD 2021

[2] G. Cacciapaglia, arXiv:2309.10098 (2023)

RGEs: from 4D to 5D

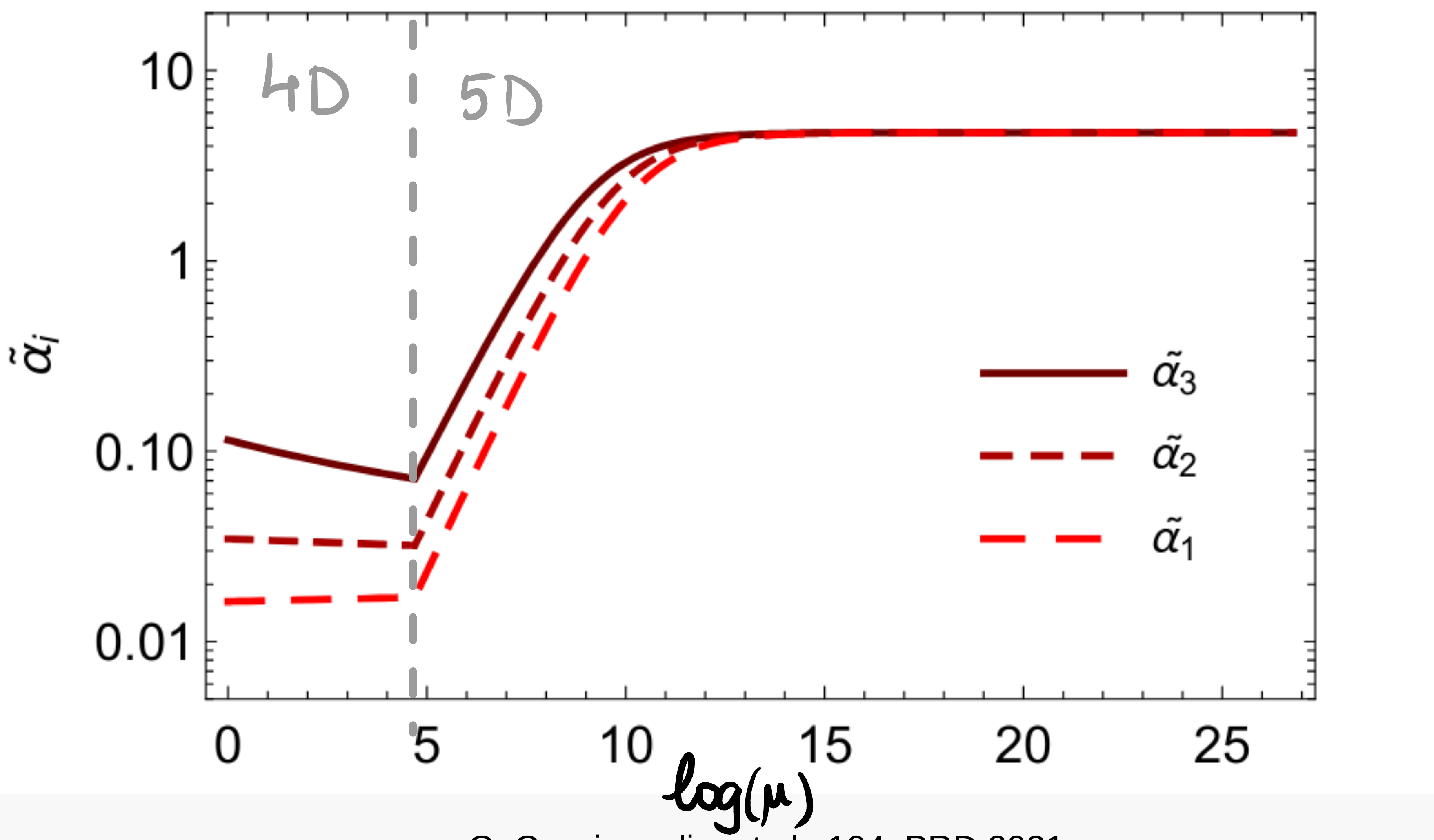
- 5D: couplings flow asymptotically towards a UV fixed point

It´s existence ⟷ good behavior in the UV

- gauge couplings

$$16\pi^2 \frac{dg_i}{dt} = b^i_{SM}\, g_i^3 + (S(t)-1) b_5^i g_i^3$$

Fixed point exists for $\mathrm{b}_5^i \leq 0$

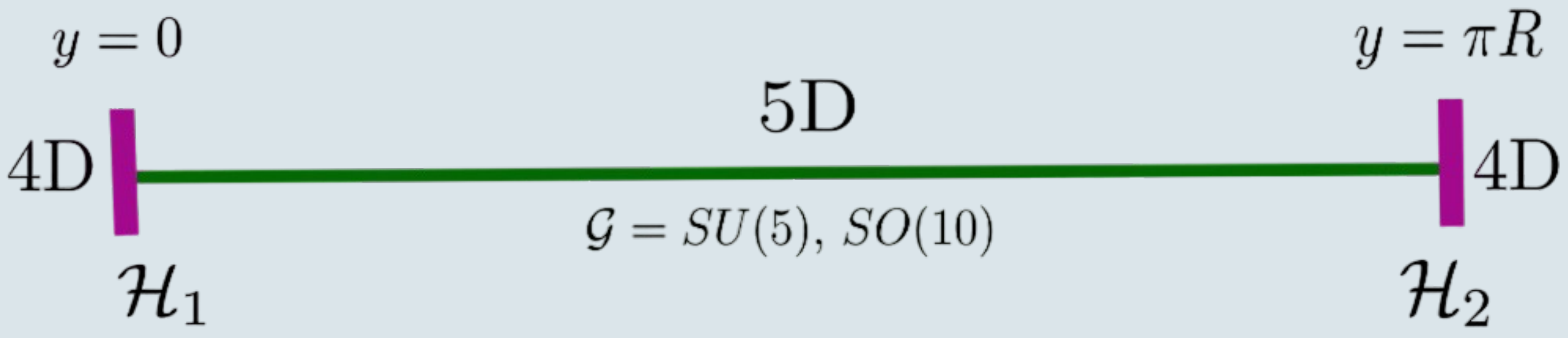


G. Cacciapaglia ,et.al., 104, PRD 2021

Assume a **theory** with couplings that flow asymptotically towards **fixed points**

+ the fixed points are perturbative

Can we **renormalize** the theory in the bulk and on the boundary?



Renormalization

Loop diagrams may be divergent (+∞)

↓

Introduce counterterms to cancel divergencies

(−∞)

↓

If the number of counterterms is finite: theory is **renormalizable**

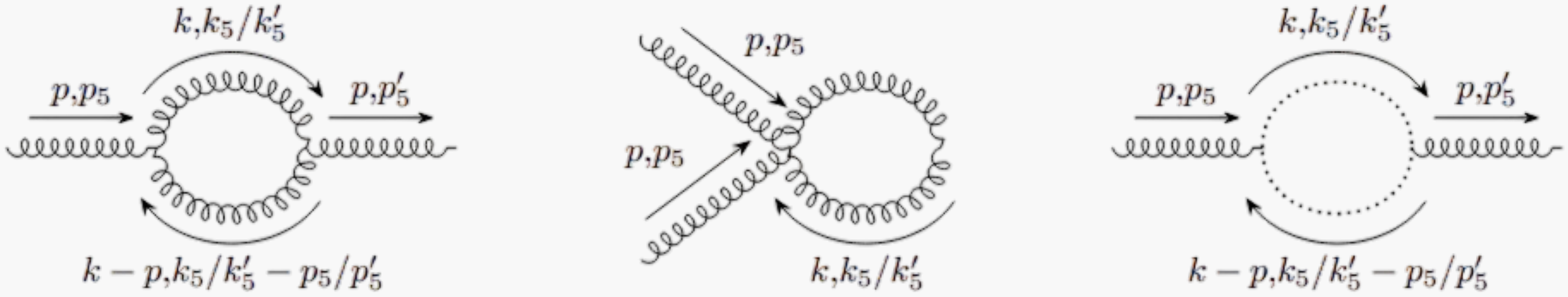
(finite)

Renormalization of 5D gauge theories

Common lore: **5D** gauge theories are **non-renormalizable** [3]

... but under certain conditions a fixed point exists → theories valid up to arbitrarily high energy scales [4]

Pure Yang Mills theory $\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} F^{MN} F_{MN}$



[3] H. Gies, Phys. Rev. D 68 (2003)

[4] T. Morris, JHEP 2005 (2005)

Renormalization of 5D gauge theories

Pure Yang Mills theory $\mathcal{L}_{\rm YM} = -\frac{1}{4} F^{MN} F_{MN}$

Bulk: 5D Yang Mills

- structure of divergencies same as in 4D
- scaling changes from 4D log to 5D linear
- **finite number of counterterms**

Boundary: Yang Mills at the fixed points

- **finite number of counterterms**
- **"magic gauge"** ($\xi = -3$): single counterterm

$$-\frac{1}{4} K \delta \left(F_{\mu\nu} F^{\mu\nu} \right)$$

! Theory is renormalizable both in the bulk and on the boundary (at one loop)

(finite number of operators to cancel divergencies)

Renormalization of 5D gauge theories

Adding scalars and fermions

- realistic models: go beyond pure Yang Mills
- Yukawa interactions: finite on the boundary [5]
- fermion and scalar 2-pt functions introduce finite number of counterterms [6]
- scalar quartic interactions also renormalizable

[5] H. Georgi, et al, Phys. Let. B 506 (2001)
[6] HC Cheng, et al., Phys. Rev. D, 66 (2002)

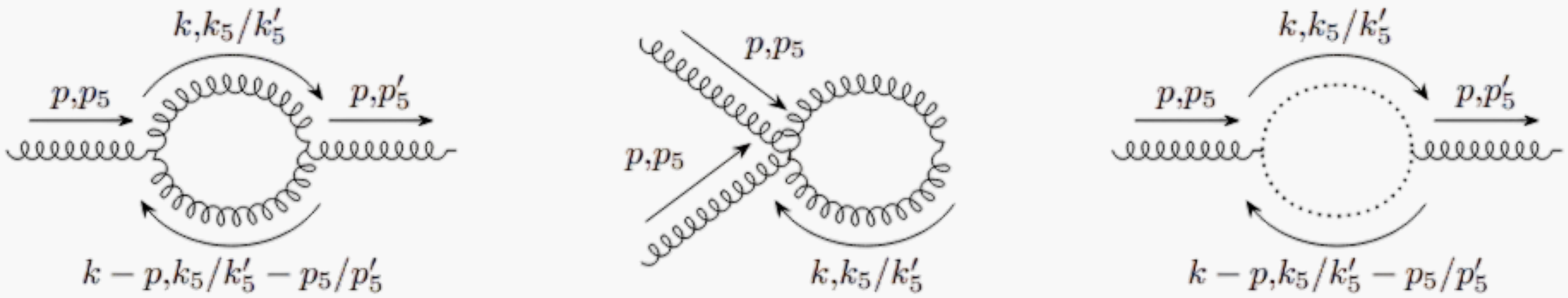
Summary and conclusions

- RGEs of couplings change in 5D: **power law running** (compared to 4D logarithmic)
- Consistency of models requires the existence of **UV fixed points**
- Fixed points ⟷ renormalizability
- Pure **Yang Mills renormalizable** at one loop (both bulk and boundary)
- Going beyond Yang Mills: also renormalizable

Backup slides

Renormalization of 5D gauge theories

Pure Yang Mills theory $\mathcal{L}_{\rm YM} = -\frac{1}{4} F^{MN} F_{MN}$



$$i\Sigma = [p_5 - \text{conserving terms}] + [p_5 - \text{non-conserving terms}]$$

Bulk: structure of divergencies same as in 4D (at one loop)

- Scaling changes from logarithmic 4D ($\sim \log \Lambda$) to 5D linear ($\sim \Lambda$)
- Same renormalization procedure as in 4D; **finite number of counterterms** are needed

Renormalization of 5D gauge theories

Pure Yang Mills theory $\mathcal{L}_{\text{YM}} = -\frac{1}{4}F^{MN}F_{MN}$

Boundary: renormalizability of a pure Yang Mills at the fixed points

→ less straightforward than bulk

Two-point function: [3,6]

$$i\Sigma = \frac{g^2}{64\pi^2}\frac{1}{\epsilon}\left[(g_{\mu\nu} - p_\mu p_\nu)\left(\frac{11}{3} - (\xi - 1)\right)C(G) + g_{\mu\nu}\frac{p_5^2 + {p'_5}^2}{2}(4 + (\xi - 1))C(G)\right]$$

To reconstruct the full counterterm on the boundary we need **higher vertex corrections** as well (3-pt and 4-pt functions)

(work in progress)

[3] G. Cacciapaglia, W. Isnard, R. Pasechnik, **AP**, (in preparation)

[6] HC Cheng, et al., Phys. Rev. D, 66 (2002)

Renormalization of 5D gauge theories

Pure Yang Mills theory $\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} F^{MN} F_{MN}$

Boundary: renormalizability of a pure Yang Mills at the fixed points

- less straightforward than bulk
- divergencies can be absorbed by a **finite number of counterterms**
- **"magic gauge"** ($\xi = -3$): coefficients of the 2-,3- and 4-pt functions are the same

  localized counterterm can be unified into a single term $-\frac{1}{4} K \delta \left(F_{\mu\nu} F^{\mu\nu}\right)$

! Theory is renormalizable both in the bulk and on the boundary (at one loop)

(finite number of operators to cancel divergencies)