

A simulation approach to evaluating the minimum detectable activity of coincidence gamma-ray spectrometers

2025-10-29

Elias Arnqvist



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Introduction

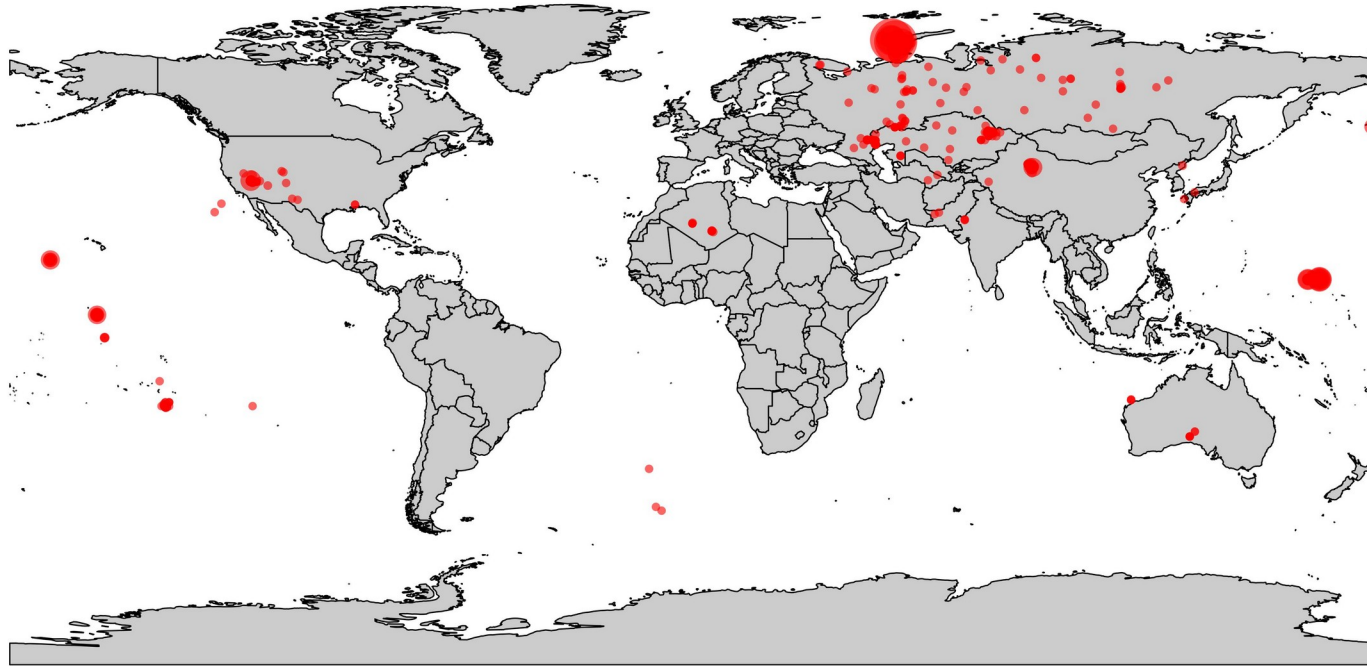
PhD student in applied nuclear physics
Uppsala University
Started 2025



Alva Myrdal Center (AMC) for Nuclear Disarmament

Collaboration with the Swedish Defence Research Agency (FOI):
Coincidence Spectrometry for Radionuclide Monitoring (CoSpeR)

Introduction



~2000 nuclear explosions

Introduction

- Comprehensive Nuclear-Test-Ban Treaty Organization (CTBTO)
- International Monitoring System (IMS)



Infrasound monitoring, Greenland

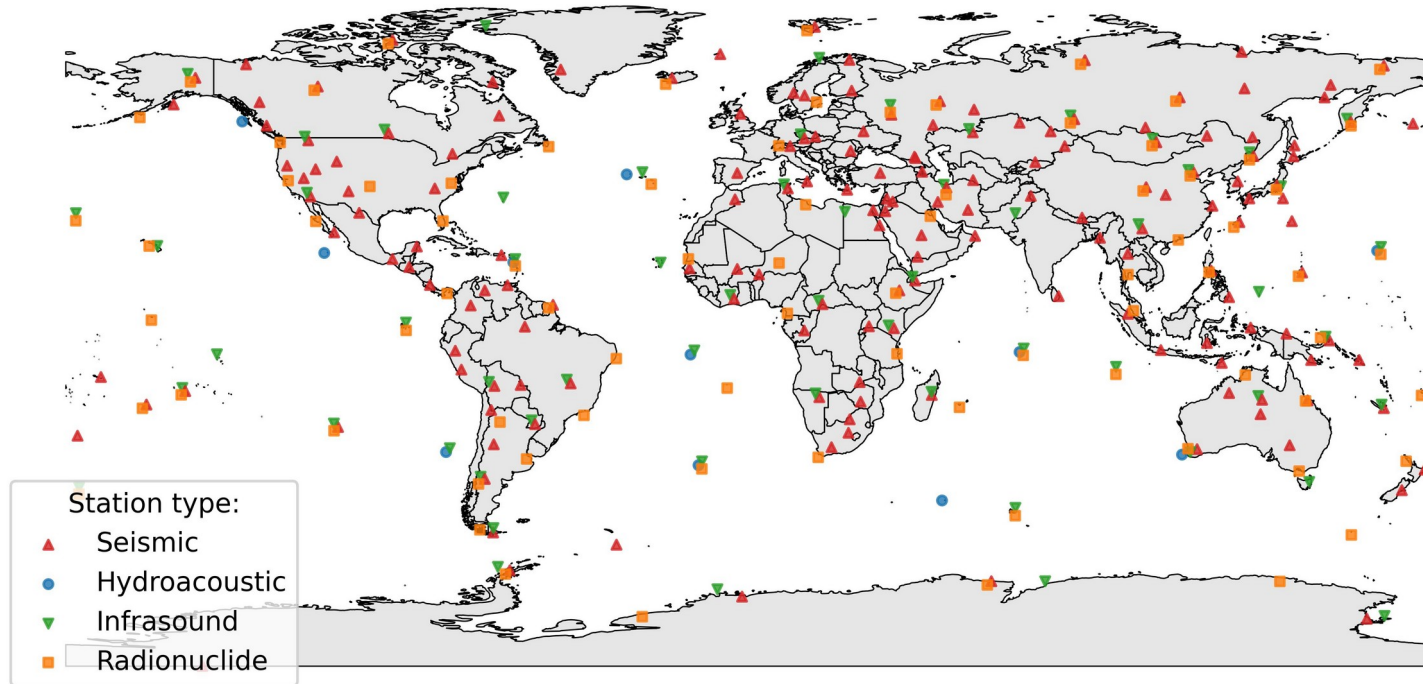


Seismic monitoring, Niger



Radionuclide monitoring,
Germany

Introduction



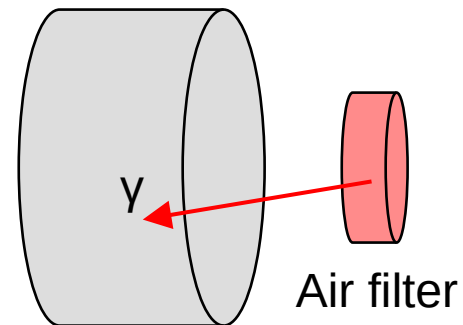
Introduction



Air sampler at UU

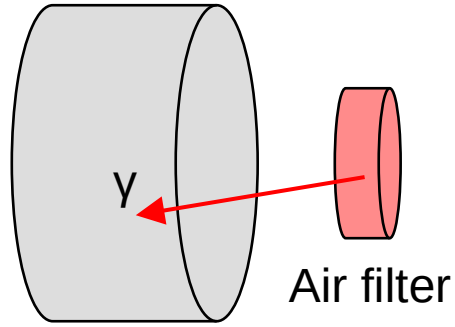


HPGe detector

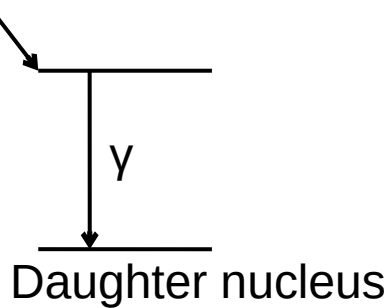


Introduction

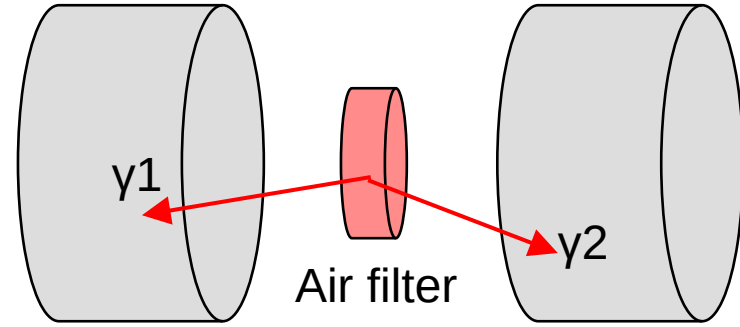
HPGe detector



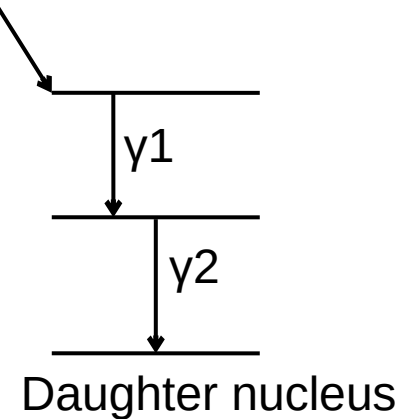
Parent nucleus



HPGe 1

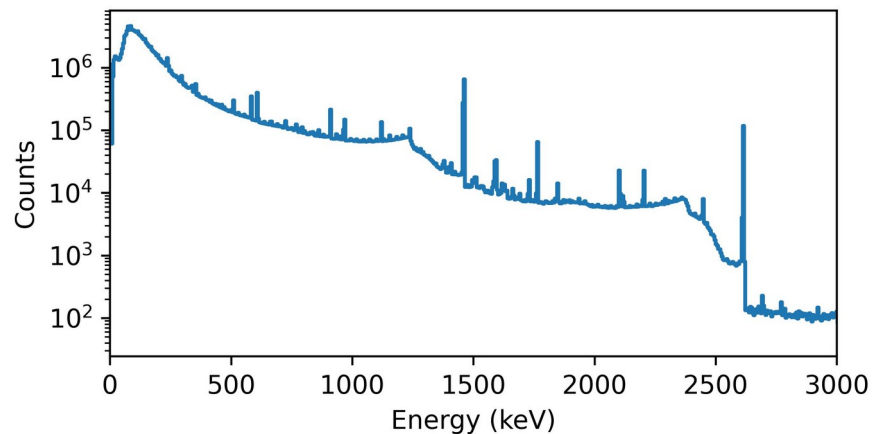
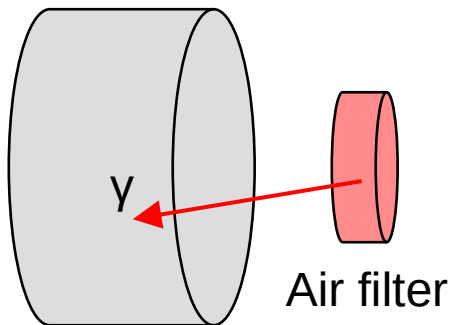


Parent nucleus

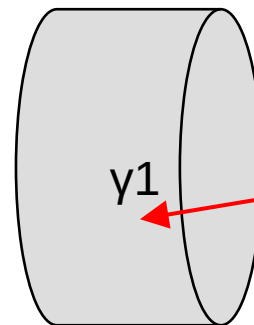


Introduction

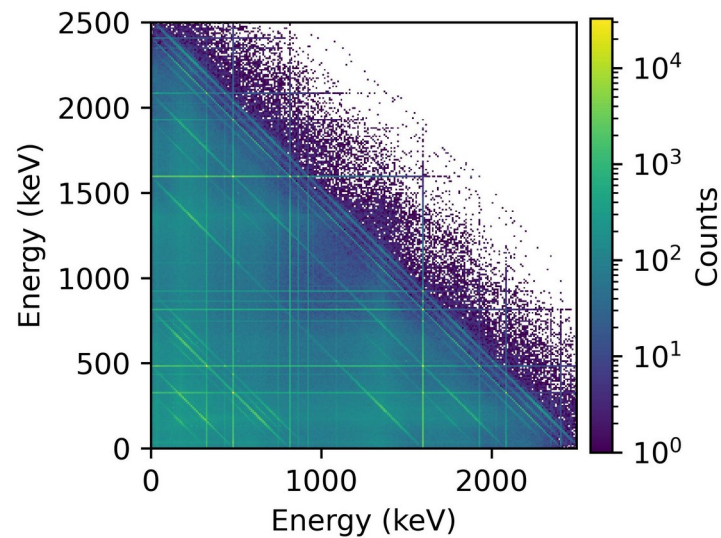
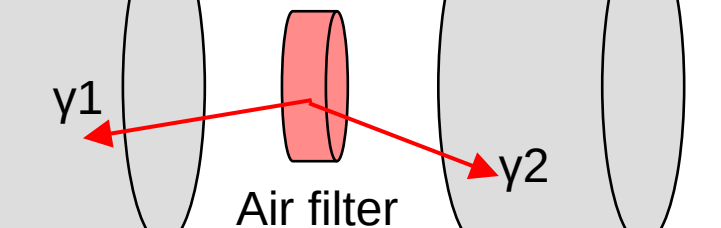
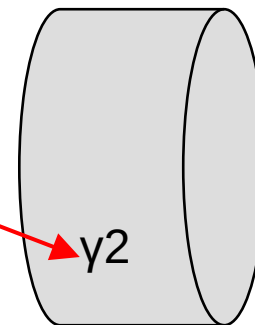
HPGe detector



HPGe 1



HPGe 2



Introduction

Minimum detectable activity:

$$MDA = \frac{L_D}{\epsilon_{\gamma\gamma} I_{\gamma\gamma} t}$$

Detection limit $L_D \approx 4.65 \sqrt{B}$ for large B

Background counts

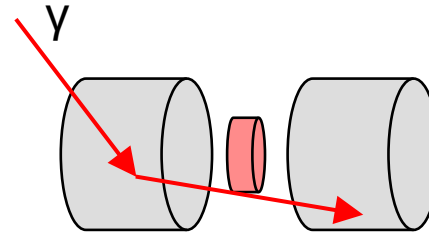
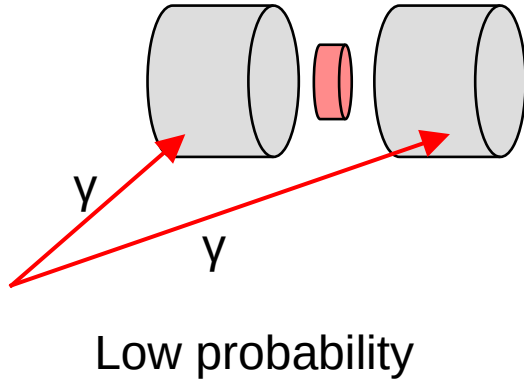
Efficiency $\epsilon_{\gamma\gamma}$ Intensity $I_{\gamma\gamma}$ Measurement time t

Goal: develop method to simulate MDA for coincidence detectors in



Simulating background radiation

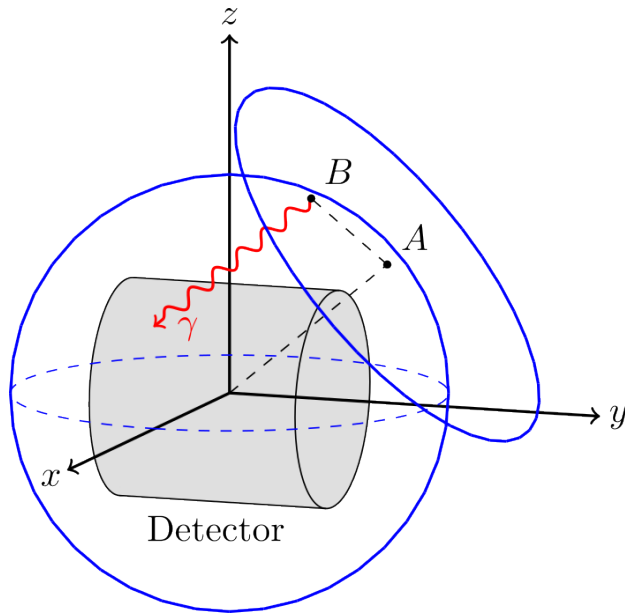
Coincidence background radiation:



- Depends on environment!
- Approximation: isotropic and uniform

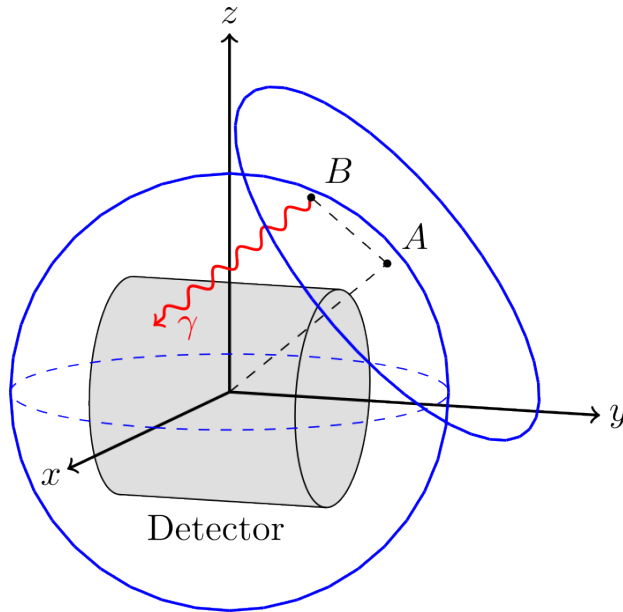
Simulating background radiation

A background model:



Simulating background radiation

A background model:



- Uniform fluence inside sphere:

$$\psi = \frac{N_\gamma}{r^2 \pi}$$

Number of gammas

Radius

- Radius can be adjusted to fit detector
- Computationally efficient
- But what energies should the gamma rays have?

Simulating background radiation

Response function:

$$c(E) = \int_0^{\infty} R(E, E') s(E') dE'$$

↑ ↑ ↑

Measured Simulated Unknown
spectrum response source spectrum
 function

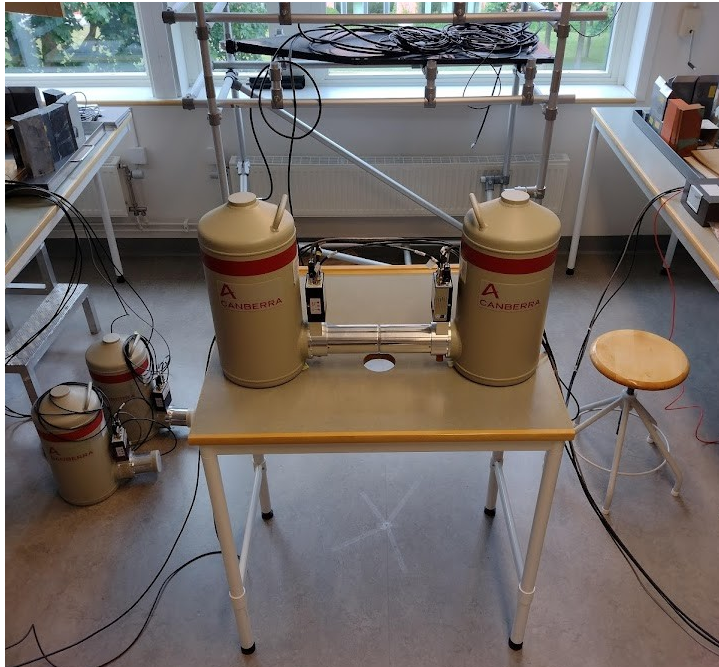
Response matrix:

$$c = R s$$

Simulating background radiation

Measuring spectra in two cases:

Unshielded at UU

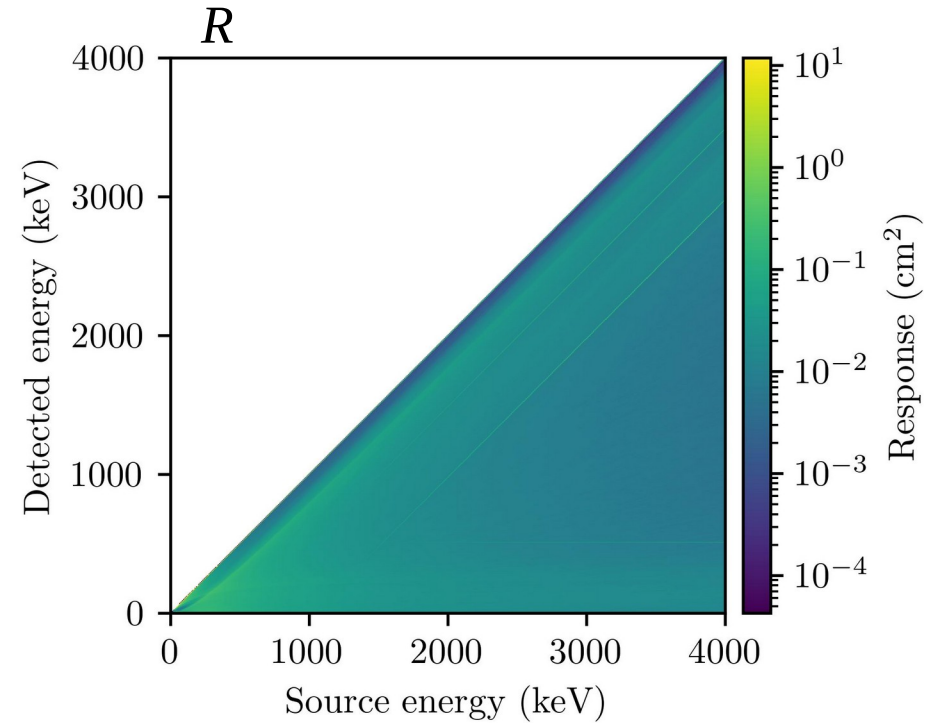
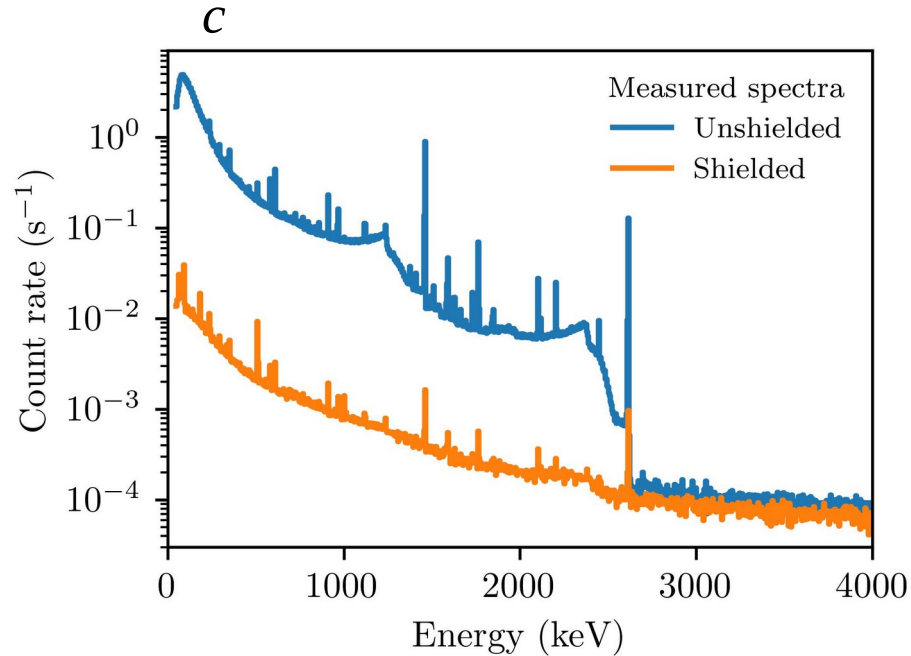


Shielded at FOI



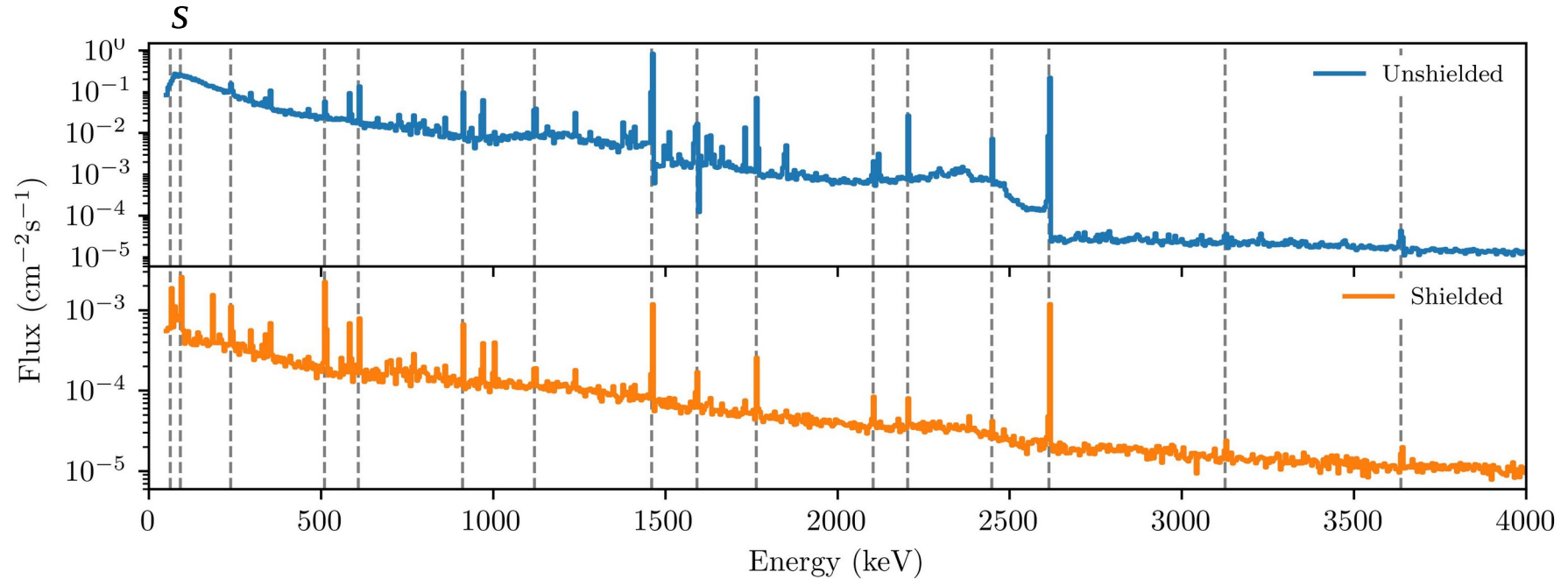
Simulating background radiation

Recall that $c = R s$



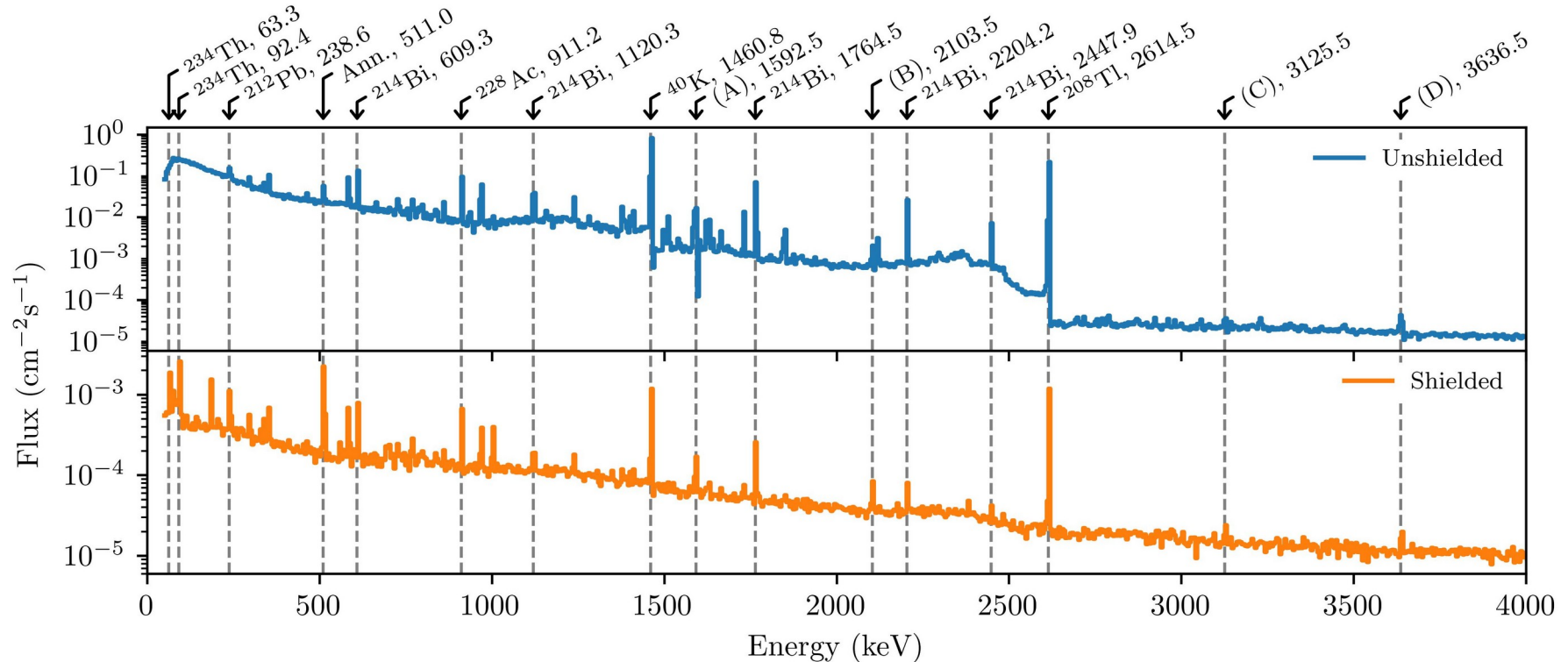
Simulating background radiation

Recall that $c = R s$



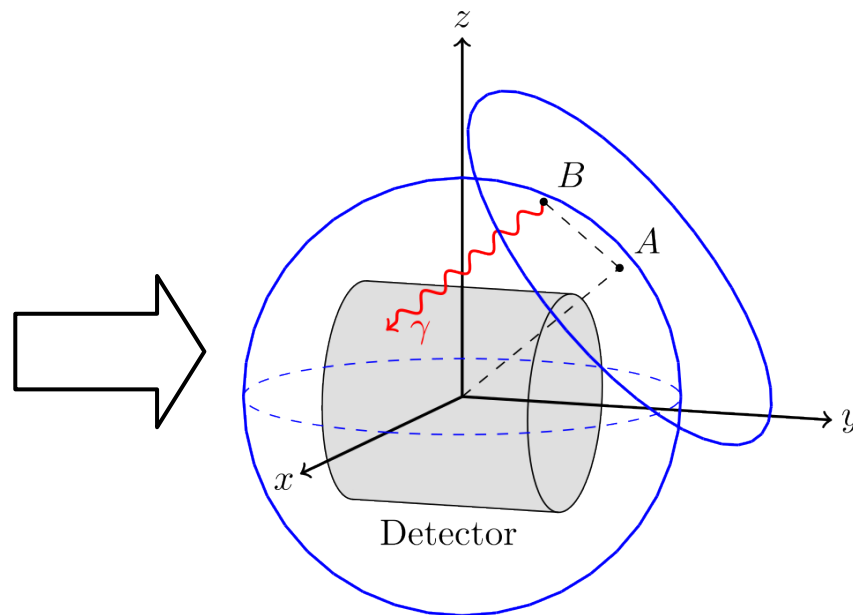
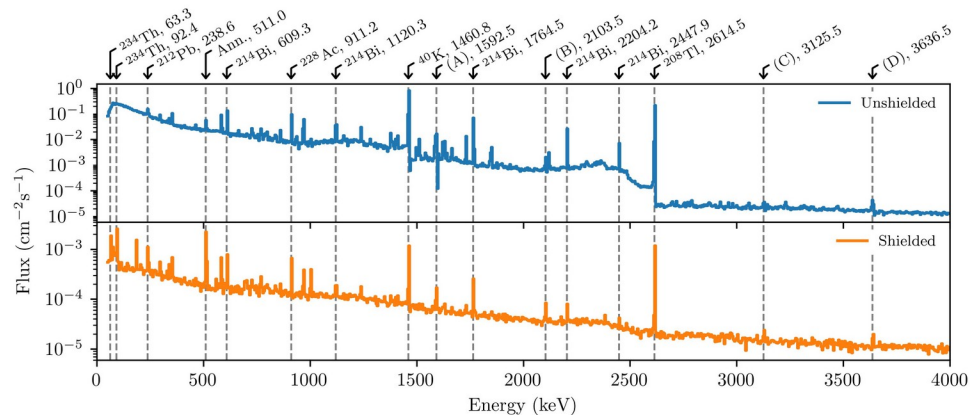
Simulating background radiation

Recall that $c = R s$



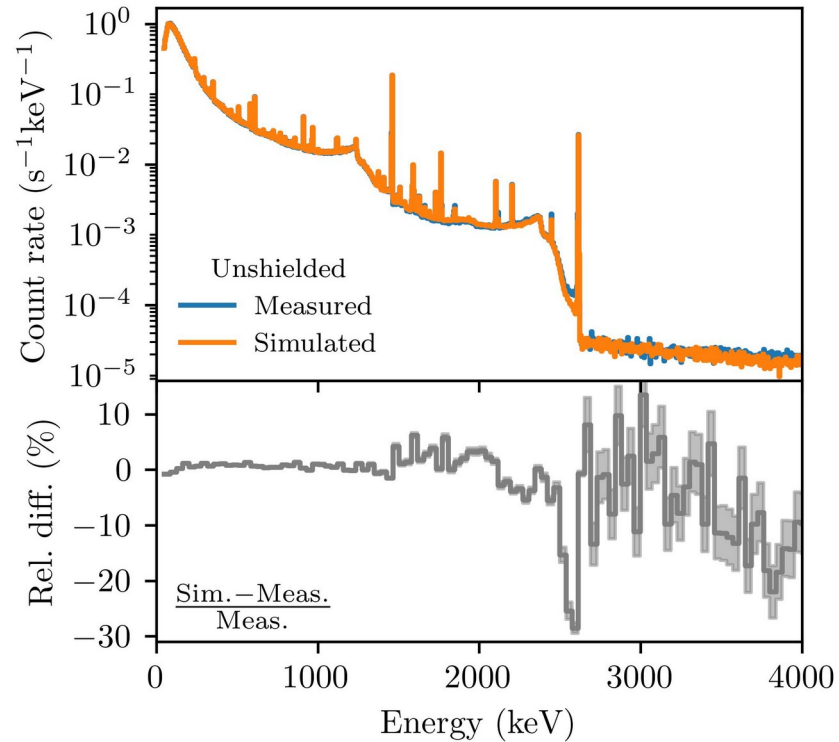
Simulating background radiation

Comparing simulations to measurements



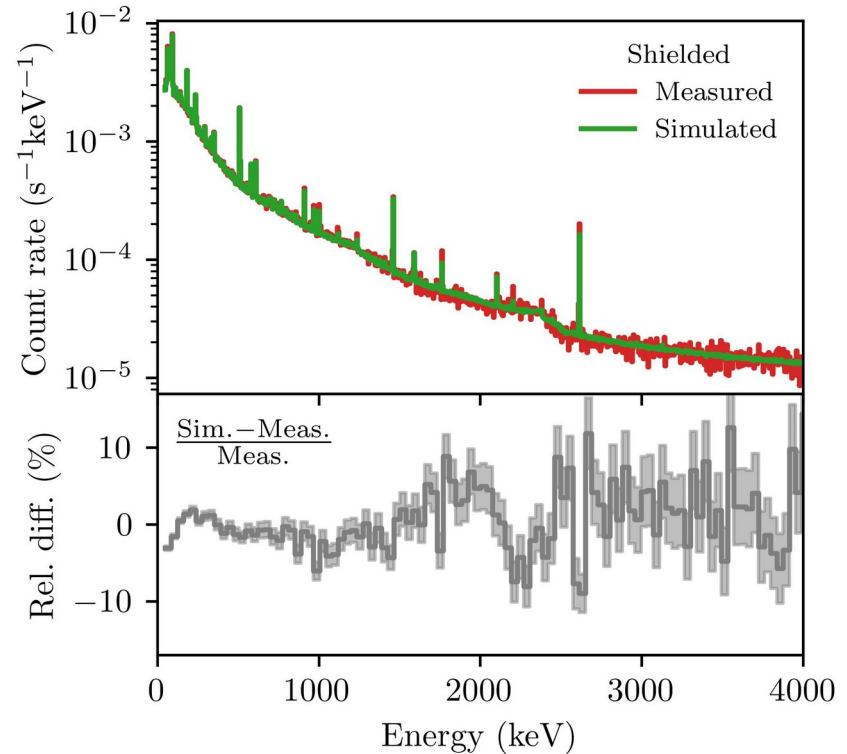
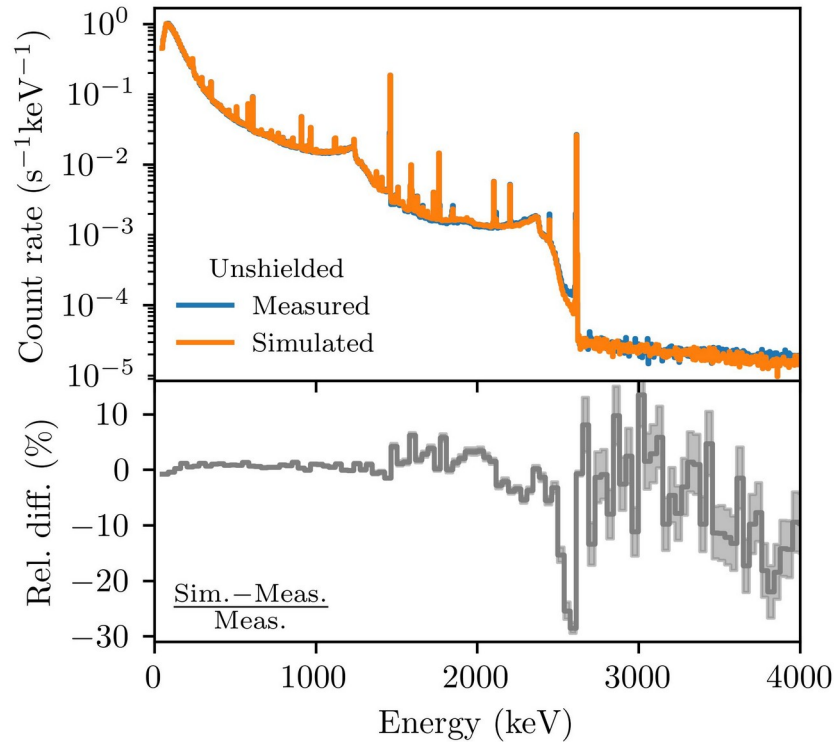
Simulating background radiation

Comparing simulations to measurements (1D)



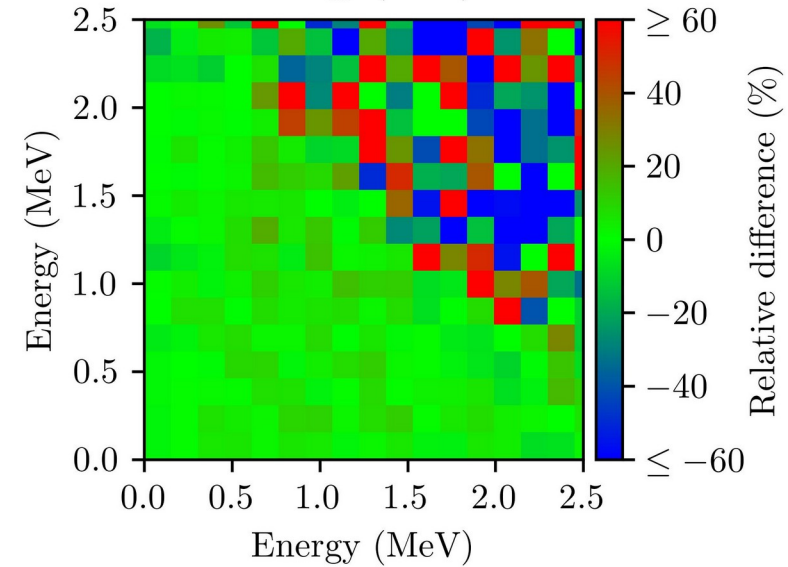
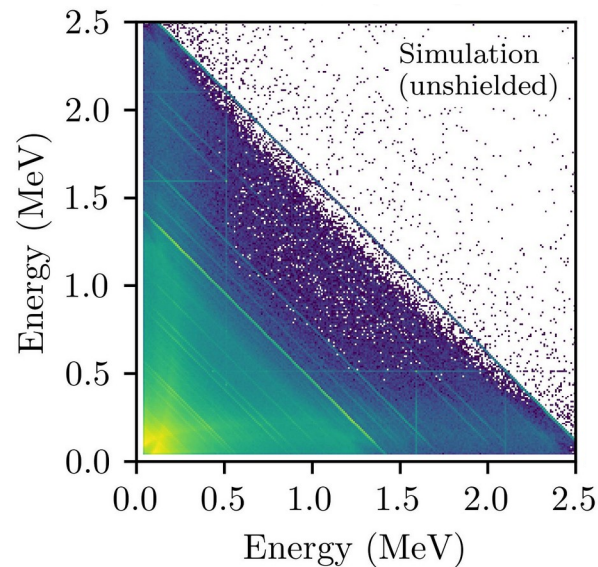
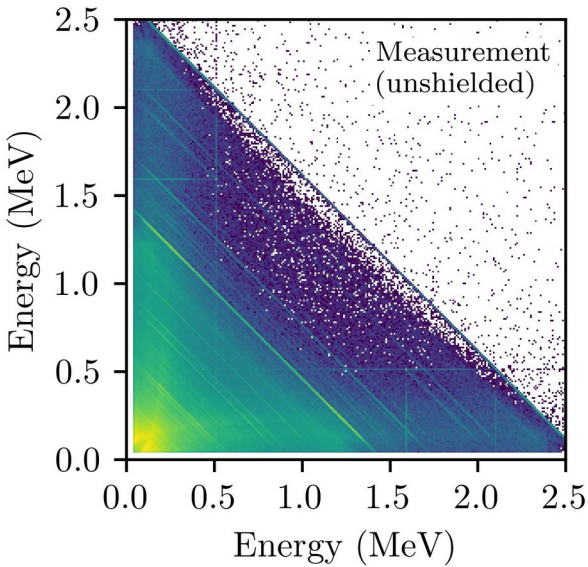
Simulating background radiation

Comparing simulations to measurements (1D)



Simulating background radiation

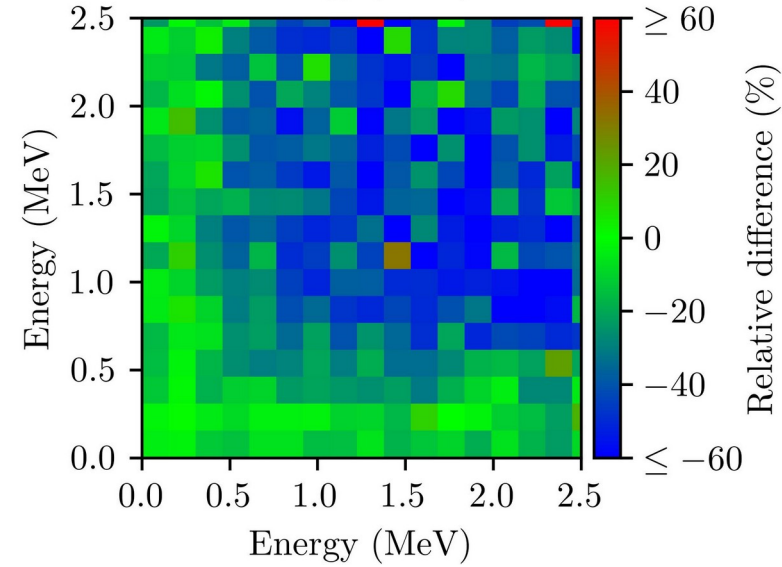
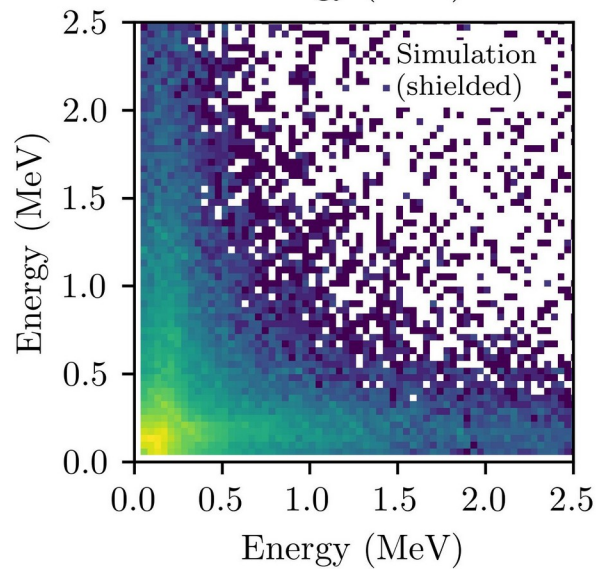
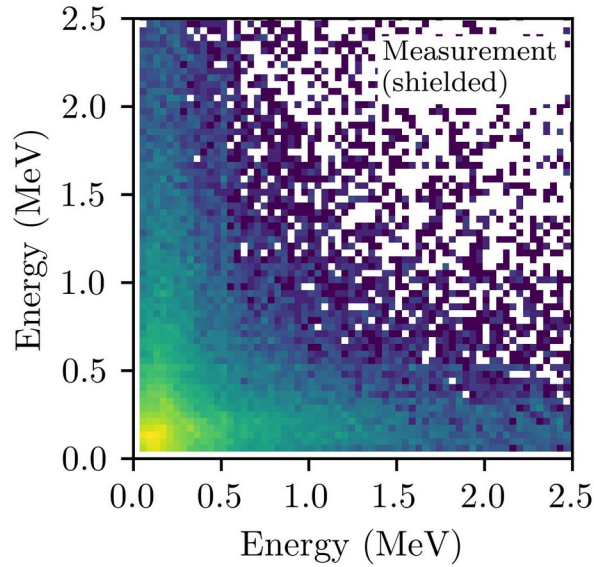
Comparing simulations to measurements (2D)



Works well for
 $E1 + E2 < 2614 \text{ keV}$

Simulating background radiation

Comparing simulations to measurements (2D)



Example

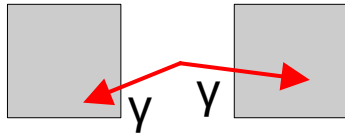
Measure La-140 point source

Two 70x70 mm cylindrical detectors, 1 mm of Al surrounding Ge, point source

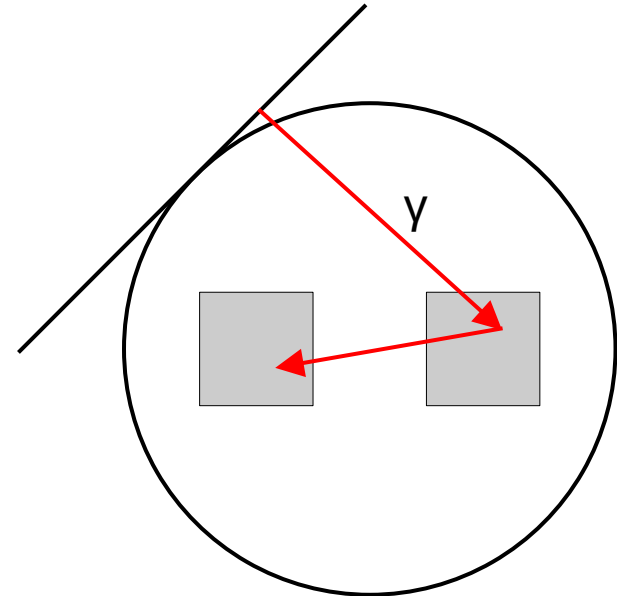
How does MDA vary with distance between the detectors?

$$MDA = \frac{L_D}{\epsilon_{\gamma\gamma} I_{\gamma\gamma} t}$$

Radionuclide simulation: $\epsilon_{\gamma\gamma} I_{\gamma\gamma}$



Background simulation: L_D

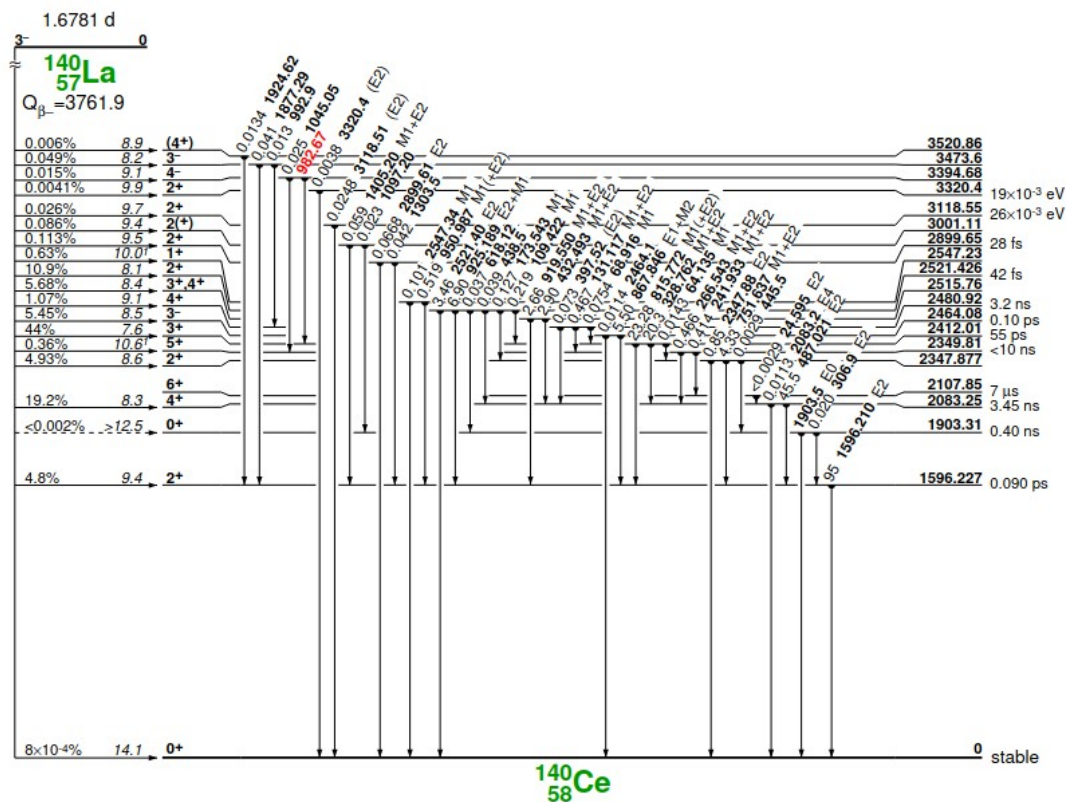


Example

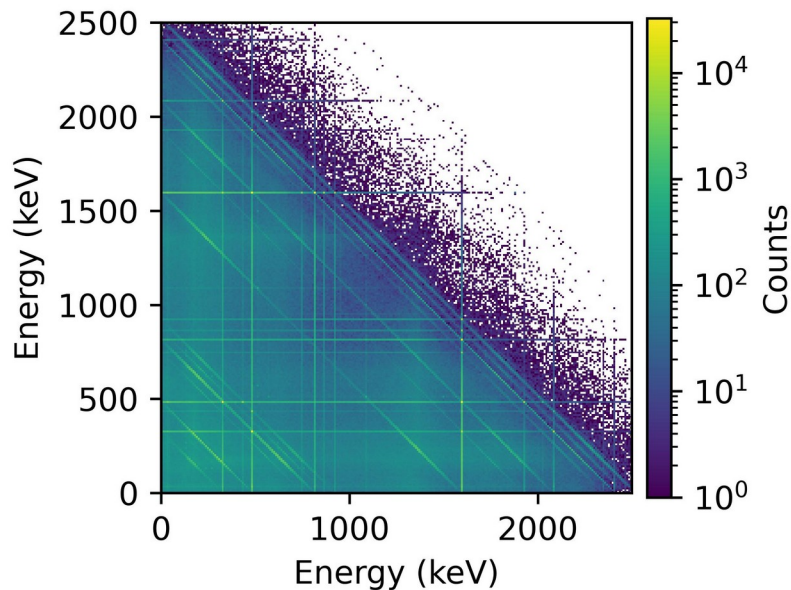


La-140

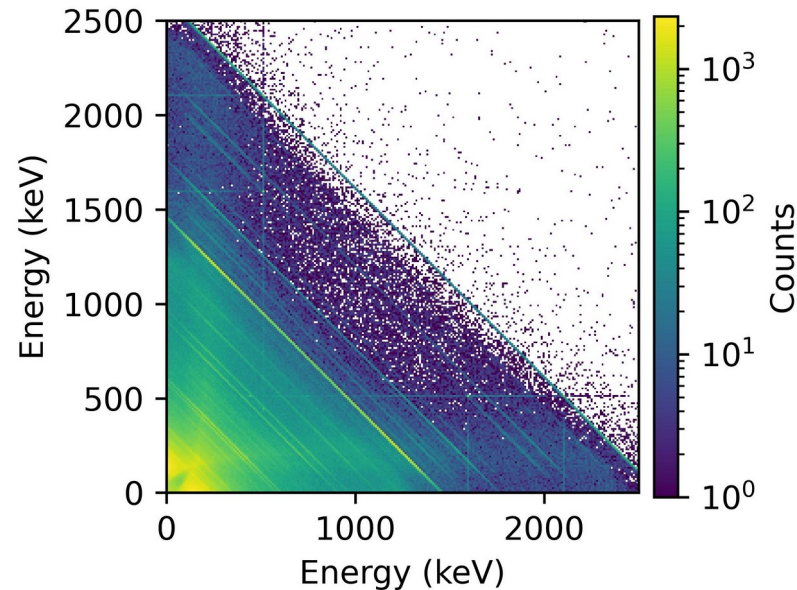
Most promising $\gamma\gamma$ -coincidences (keV): [329, 487]; [816, 1596]; [487, 1596]



Example



Radionuclide simulation



Background simulation

$$MDA = \frac{L_D}{\varepsilon_{YY} I_{YY} t}$$

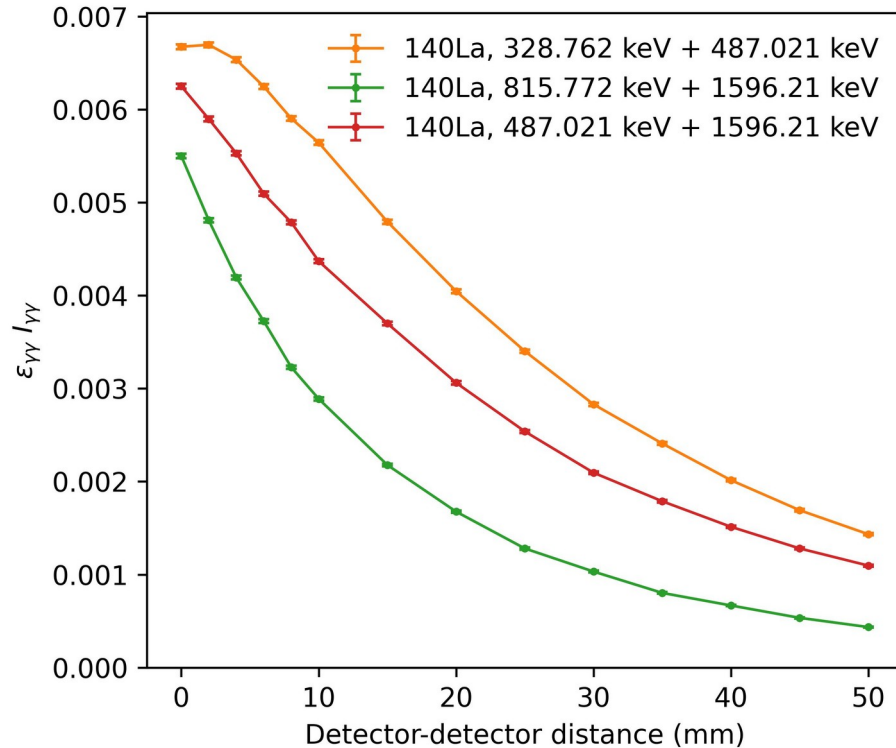
Diagram illustrating the Minimum Detectable Activity (MDA) formula. The term $\varepsilon_{YY} I_{YY}$ is associated with the Radionuclide simulation, and L_D is associated with the Background simulation. Arrows point from these labels to their respective terms in the formula.

Example

$$MDA = \frac{L_D}{\varepsilon_{\gamma\gamma} I_{\gamma\gamma} t}$$


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(La-140, $t = 1$ day, no decay correction, unshielded environment)

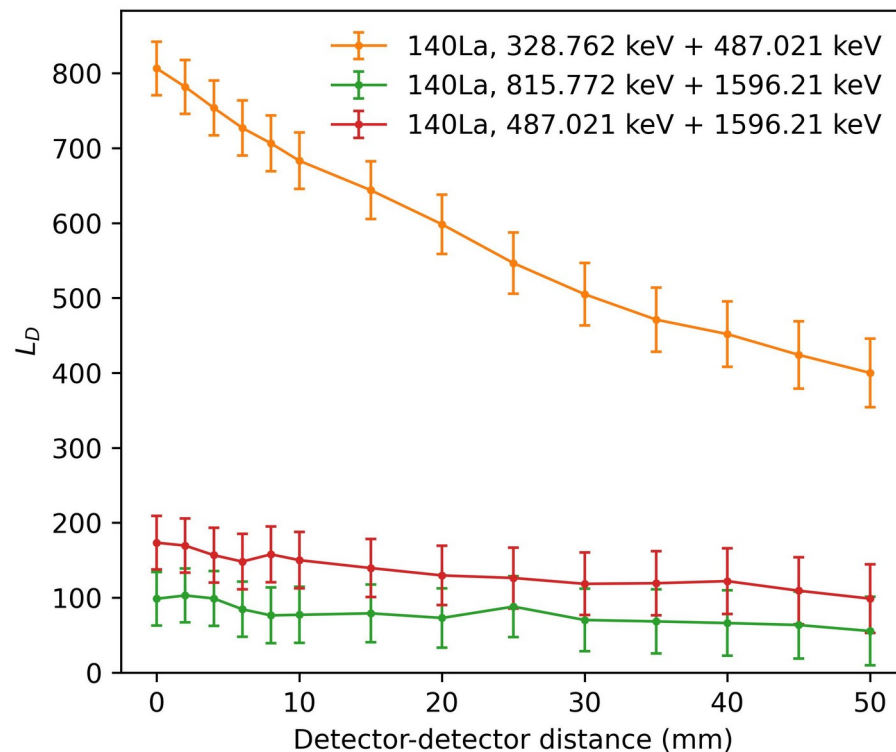
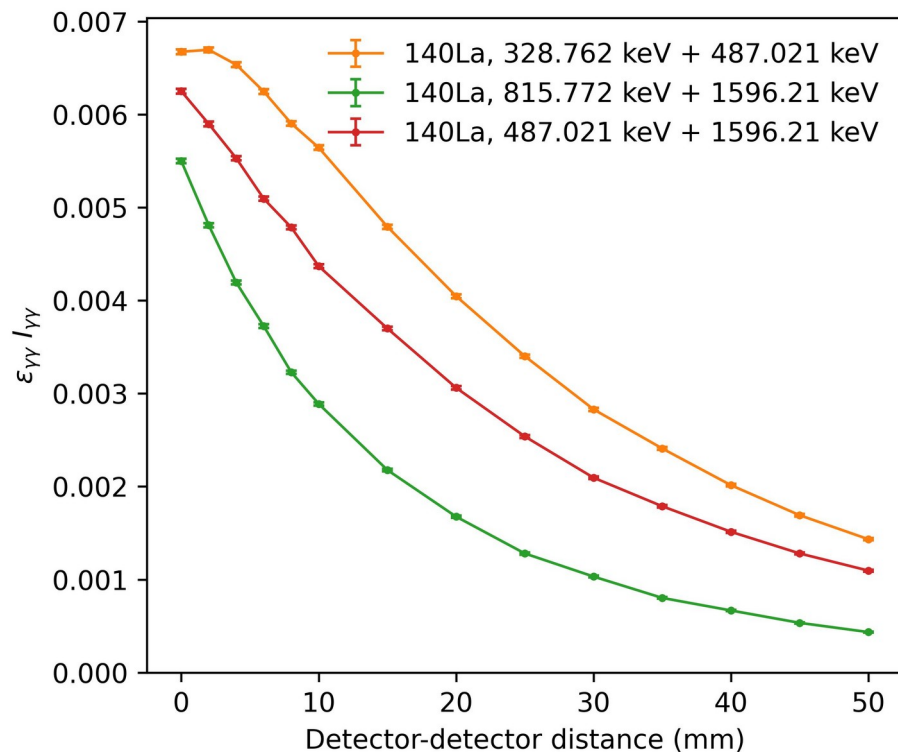


Example

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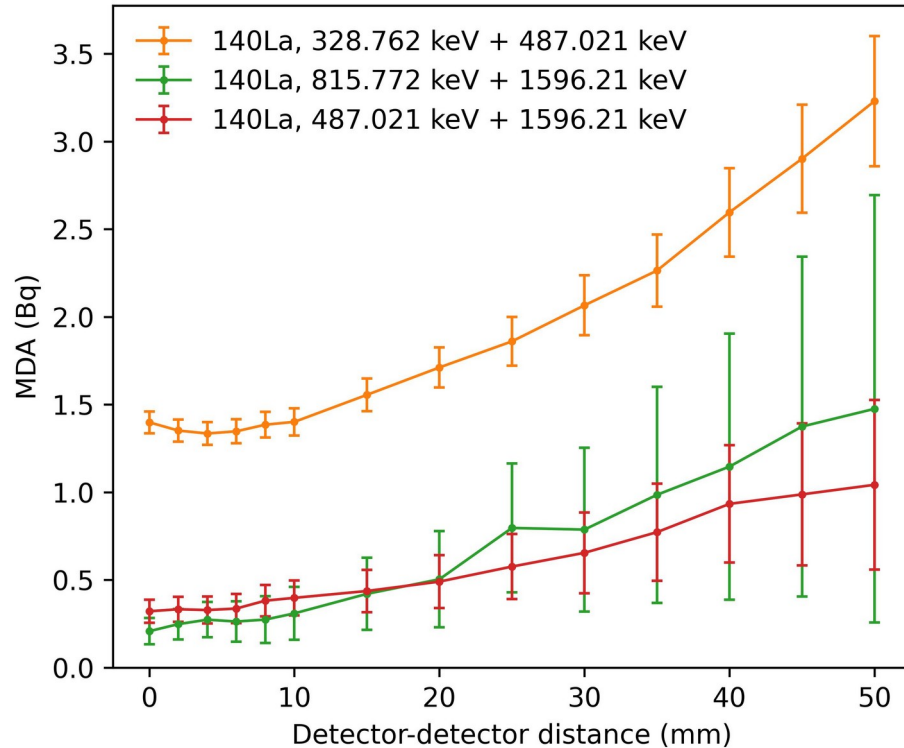


Example

$$MDA = \frac{L_D}{\varepsilon_{\gamma\gamma} I_{\gamma\gamma} t}$$


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(La-140, $t = 1$ day, no decay correction, unshielded environment)

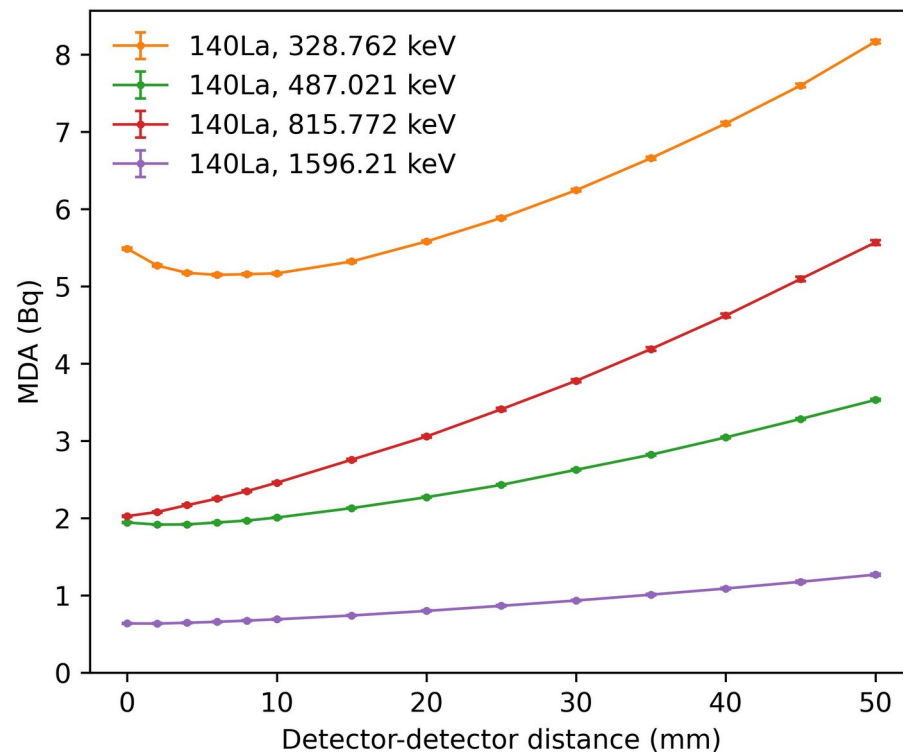
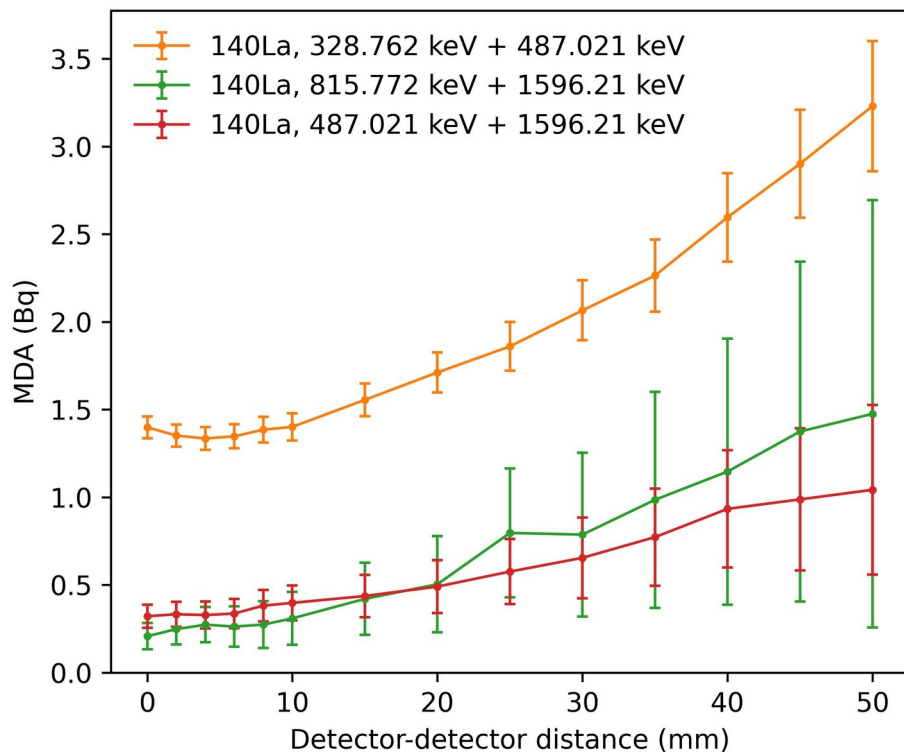


Example

$$MDA = \frac{L_D}{\varepsilon_{YY} I_{YY} t}$$


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(La-140, $t = 1$ day, no decay correction, unshielded environment)

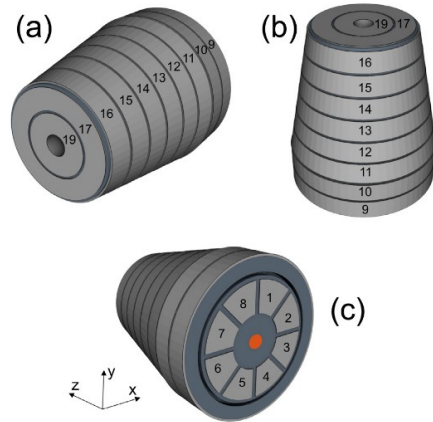


Next steps

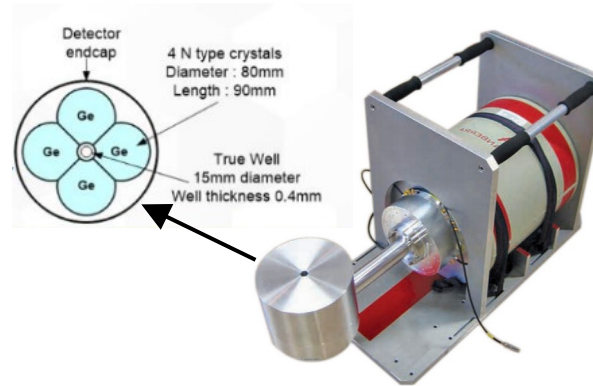
- More complex detectors



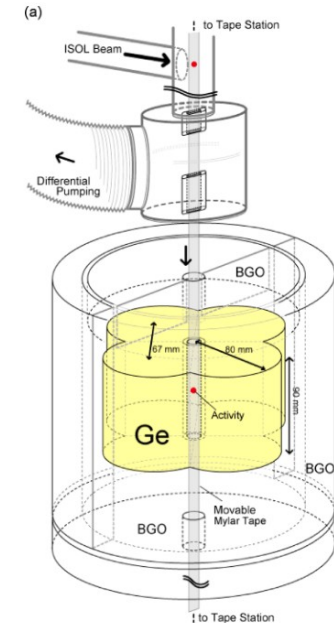
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F.J. Pearce et al.
10.1016/j.nima.2021.166044



Mirion Technologies



H. Hayashi et al.
10.1016/j.nima.2014.02.012

Next steps

- More complex detectors
- Other radionuclides
 - Candidates: Ba-140/La-140, Mo-99, Te-132/I-132, I-131, I-133, I-135

Next steps

- More complex detectors
- Other radionuclides
 - Candidates: Ba-140/La-140, Mo-99, Te-132/I-132, I-131, I-133, I-135
- What detector achieves the lowest MDA?



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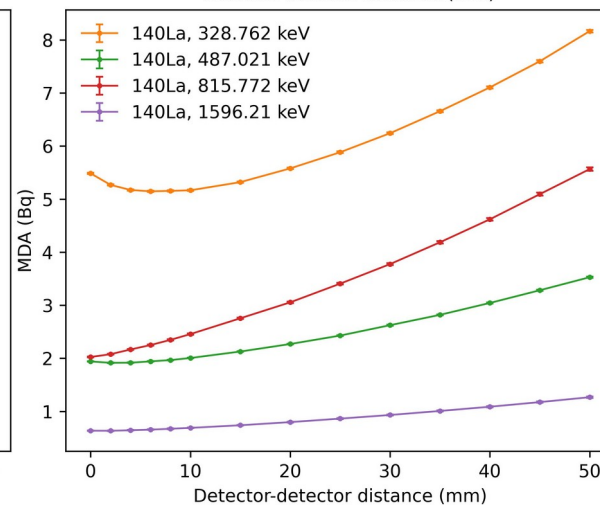
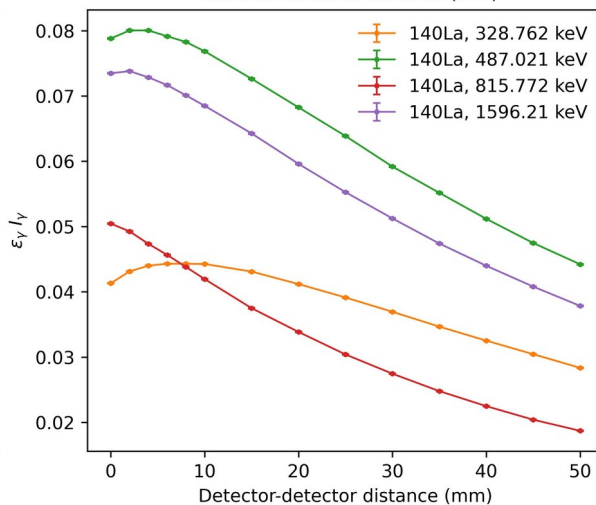
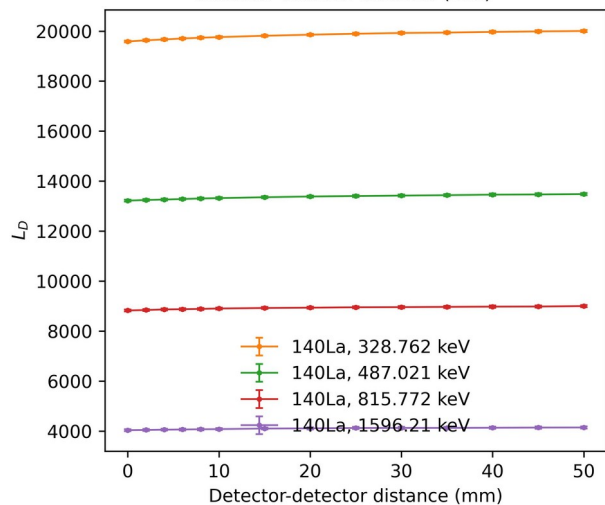
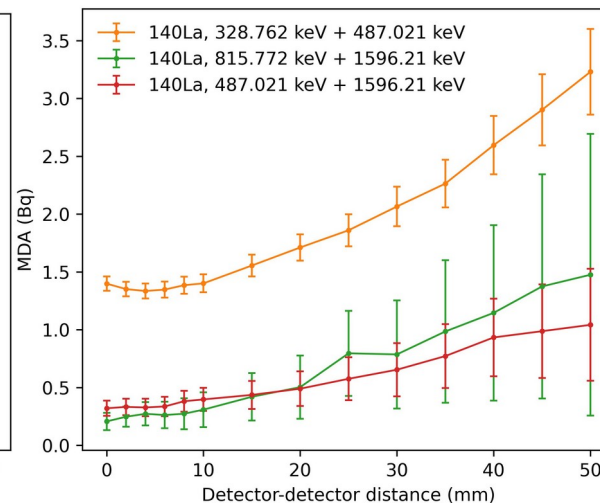
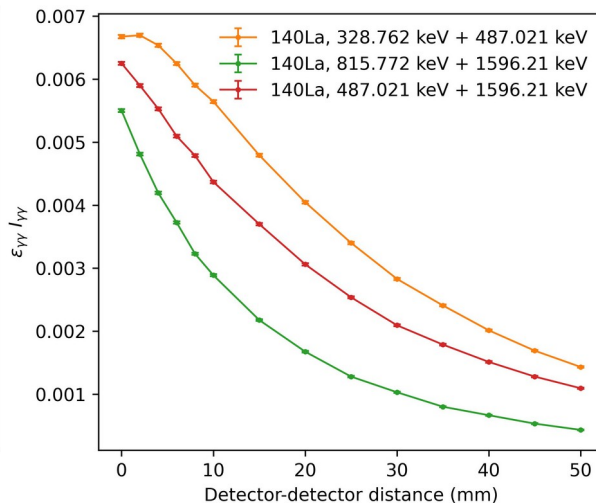
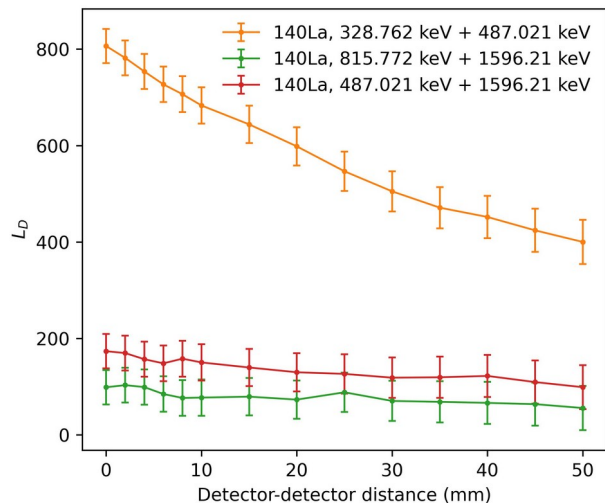
The end

Backup slides

Example: simple cylindrical detectors



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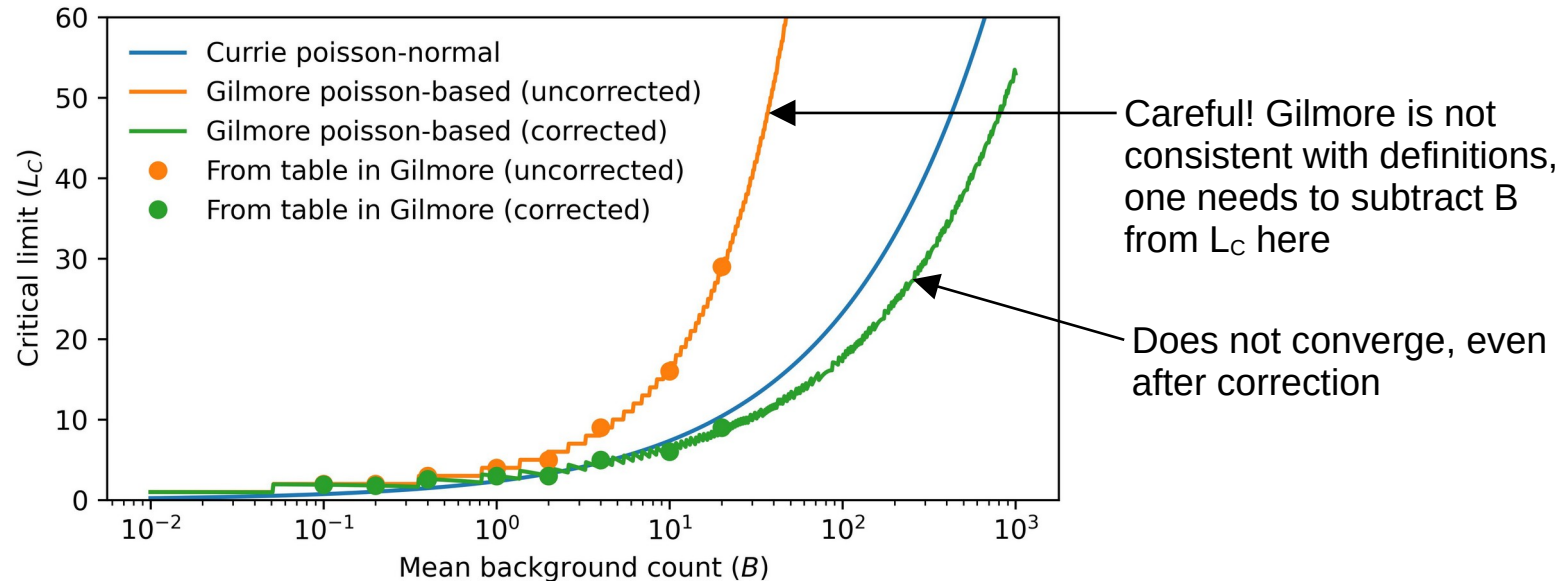


The alternatives

- Currie formula (Poisson-normal) $L_C = k_\alpha \sqrt{2B}$ $L_D = k_\alpha^2 + 2k_\alpha \sqrt{2B}$
- Stapleton approximation $L_C = \frac{k_\alpha^2}{2} + k_\alpha^2 \sqrt{2(B+0.4)}$ $L_D = L_C + \frac{k_\alpha^2}{2} + k_\alpha \sqrt{\frac{k_\alpha^2}{4} + L_C + 2B}$
- ISO standard 19929 (MDA directly) $MDA = w \frac{k^2 + 2k \sqrt{2B}}{1 - k^2 \text{var}(w)}$ $w = \frac{1}{\varepsilon I t}$
- Poisson definition (as in Gilmore)
- Poisson definition (“the correct way”)

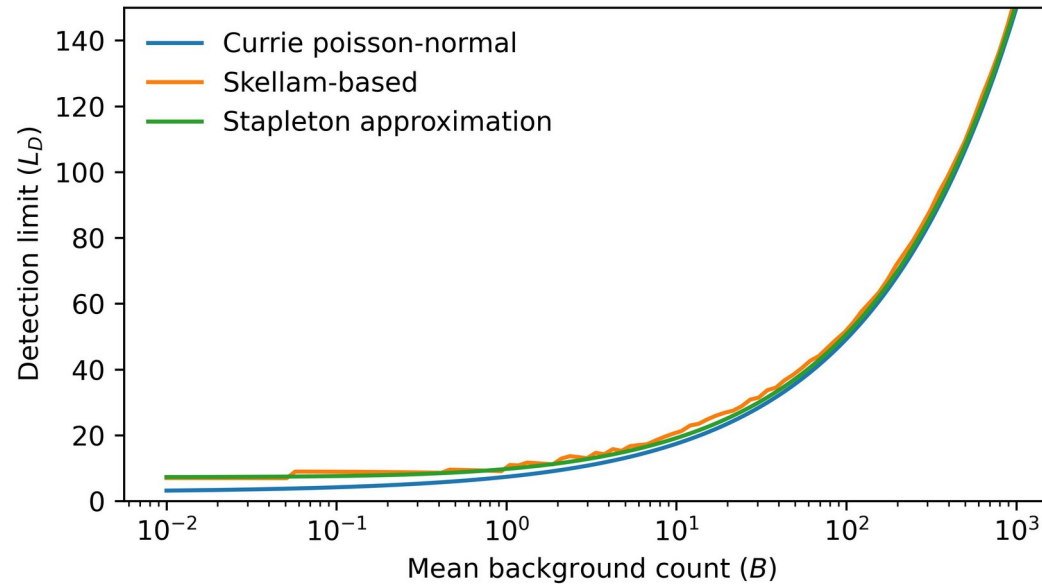
Gilmore definitions

- Gilmore uses Poisson-normal (Currie) and Poisson definition
- They do not converge for high counts!



Detection limits

- More or less the same



Radionuclides



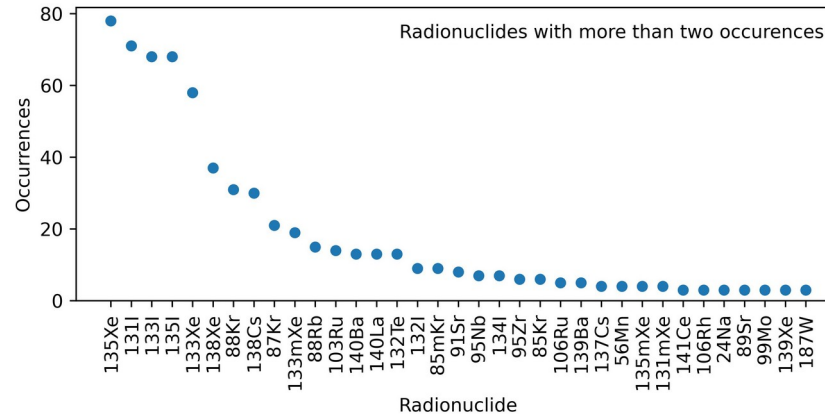
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Which radionuclides?

- Coincident gamma emission
- Good half-life
- Produced in tests
- Beneficial if well-known

Shortlist:

- Ba-140/La-140
- Te-132/I-132
- Mo-99
- I-131
- I-133
- I-135



DOE/NV-317 (Rev. 1)
UC-702

DOE/NV-317 (Rev. 1)
UC-702

RADIOLOGICAL EFFLUENTS RELEASED FROM U.S. CONTINENTAL TESTS 1961 THROUGH 1992



AUGUST 1996

UNITED STATES DEPARTMENT OF ENERGY
NEVADA OPERATIONS OFFICE

Unfolding background spectrum

Implementation in Geant4:

