Response functions from a Chebyshev expansion

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Chalmers University of Technology, Göteborg, 28/10/2025

Neutrinos and nuclei

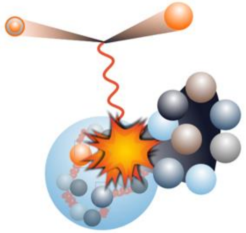
Are elusive

oscillate

Maybe their own antiparticle

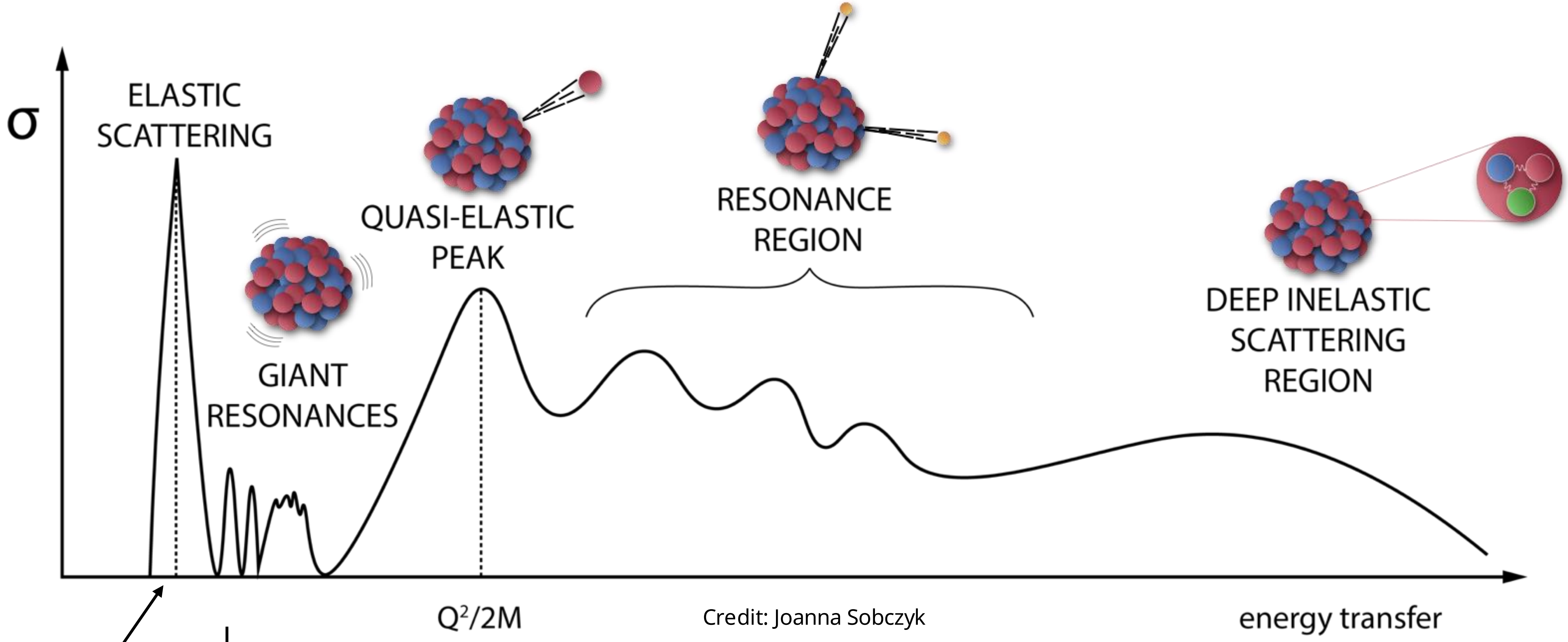
We need to understand their interactions with nuclei

to understand neutrino properties



Credit: Symmetry Magazine, ph.ed.ac.uk

Neutrino-Nucleus Scattering



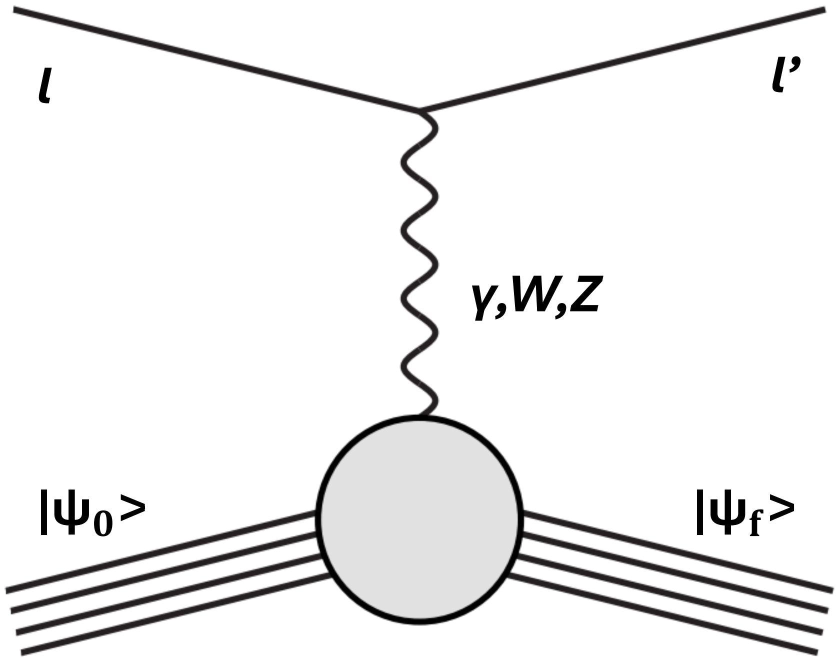
Credit: Joanna Sobczyk

Inelastic scattering

Elastic scattering: CEvNS	**Supernova neutrinos**	**Long-baseline experiments (HyperK, DUNE)**

Lepton-Nucleus Scattering

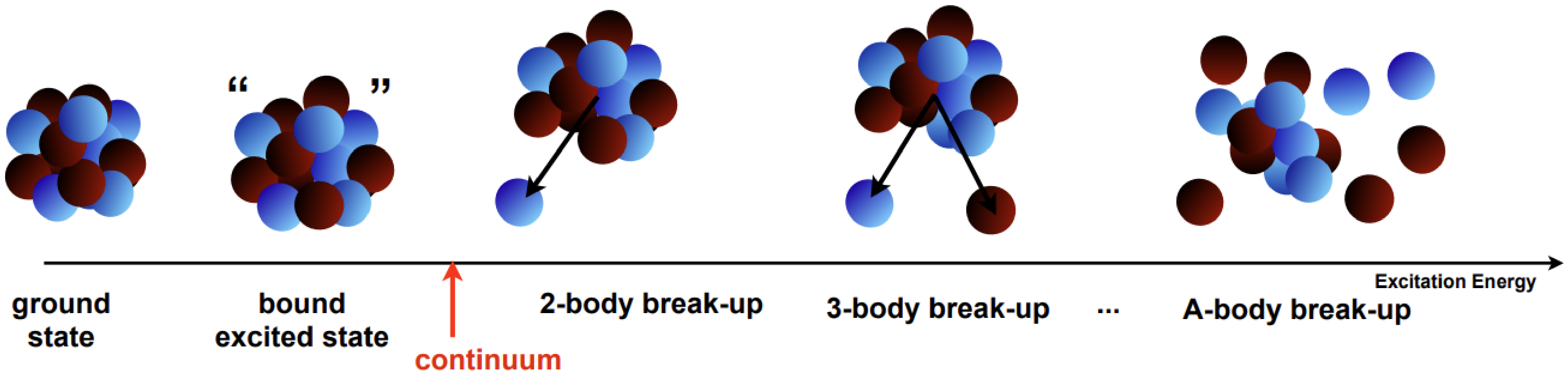
- Under some assumptions cross section factorizes
- Nuclear part is similar for electron-nucleus (neutrino-nucleus) scattering



$$\sigma \propto \underbrace{L^{\mu\nu}}_{\text{lepton tensor}} \; \underbrace{R_{\mu\nu}}_{\text{nuclear responses}}$$

Nuclear Response Functions

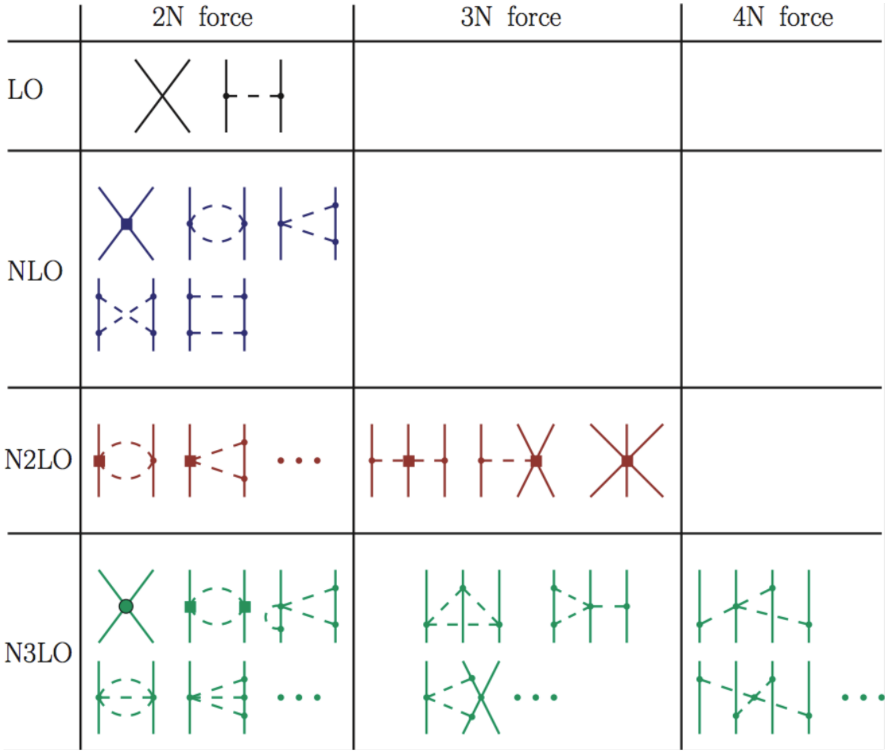
$$R_\alpha(\mathbf{q},\omega) = \sum\!\!\!\!\!\!\!\int_f \left| \langle \Psi_f | O_\alpha(\mathbf{q}) | \Psi_0 \rangle \right|^2 \delta\left(E_f - E_0 - \omega\right)$$



Bacca, Eur. Phys. J Plus 131, 107 (2016)

Hamiltonian & currents from EFT

- QCD is intractable at nuclear energies for > 1 nucleon
- Exploit approximate chiral symmetry of low energy QCD ($m_u \cong m_d \cong 0$)
- Construct a Lagrangian of interacting nucleons, pions (and deltas) constrained by QCD



$$J = \sum_i j_i + \boxed{\sum_{i<j} j_{ij}} + \ldots$$

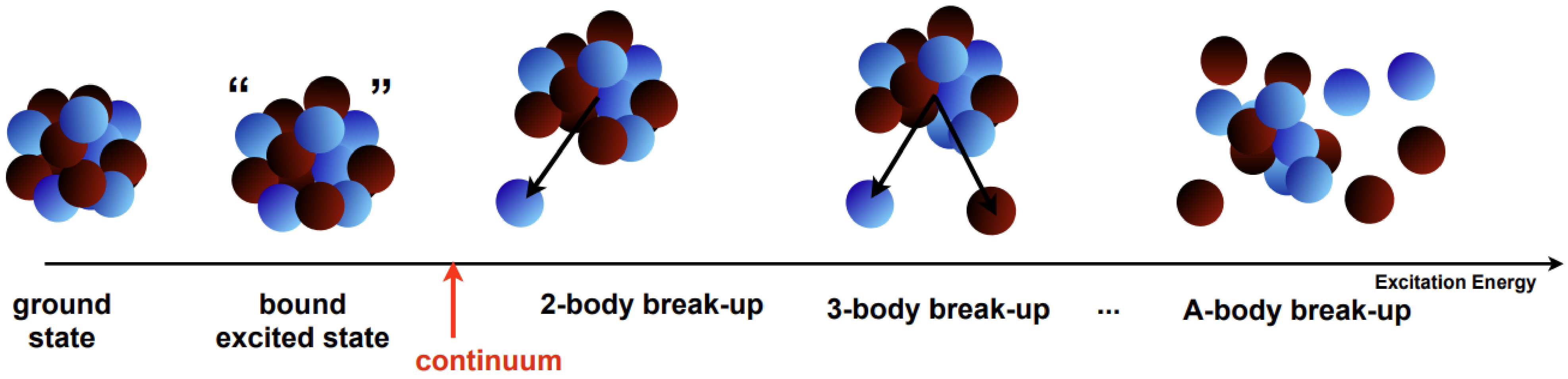
Many body problem

- Solve $H|\Psi\rangle = E|\Psi\rangle$

- Finite dimensional expansion in bound basis (harmonic oscillator)

$$|\Psi_0\rangle = \sum_k c_k \, |n_k l_k \ldots.\rangle \qquad \hat{O}_{ij} = \langle n_i l_i \ldots | \, \hat{O} \, | n_j l_j \ldots \rangle$$

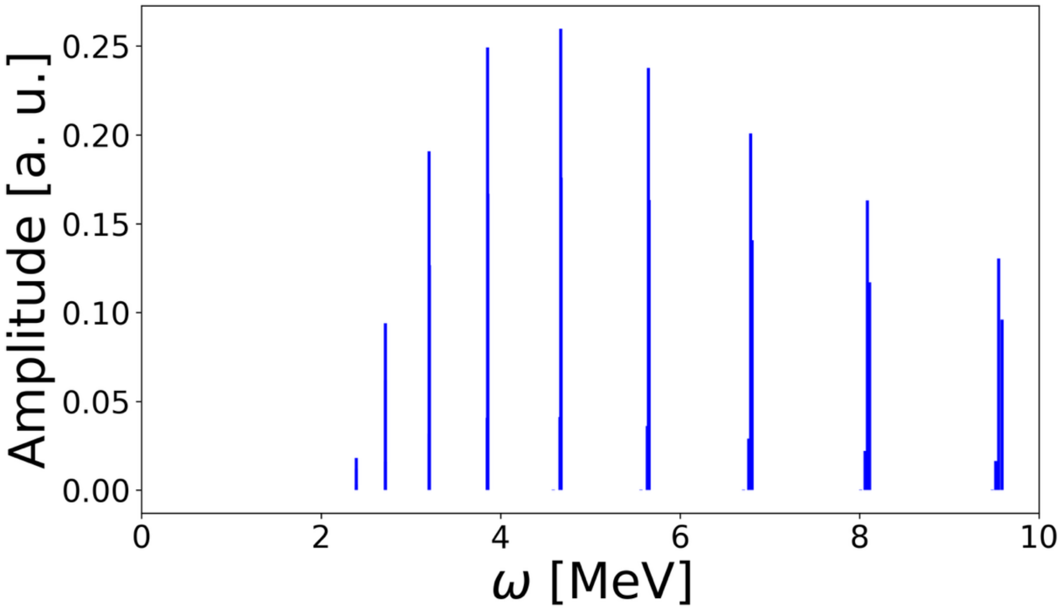
Many body problem



→Finite number of eigenvalues of H and “pseudo continuum states” with bound state boundary conditions

Response in bound state approaches

$$R_\alpha(\mathbf{q},\omega) = \sum\!\!\!\!\!\!\int_f \left| \langle \Psi_f | O_\alpha(\mathbf{q}) | \Psi_0 \rangle \right|^2 \delta\left(E_f - E_0 - \omega\right)$$



- Unphysical discrete response
- Diagonalizing H computationally prohibitive

Integral transforms

- Can avoid the final states and the discrete nature of the response

$$\Phi(\sigma) = \int d\omega\, K(\sigma, \omega) R(\omega)$$
$$= \langle \Psi_0 | \, O_\alpha^\dagger(\mathbf{q}) K(\sigma, H - E_0) O_\alpha(\mathbf{q}) \, | \Psi_0 \rangle$$

- Kernels are usually some representation of delta function

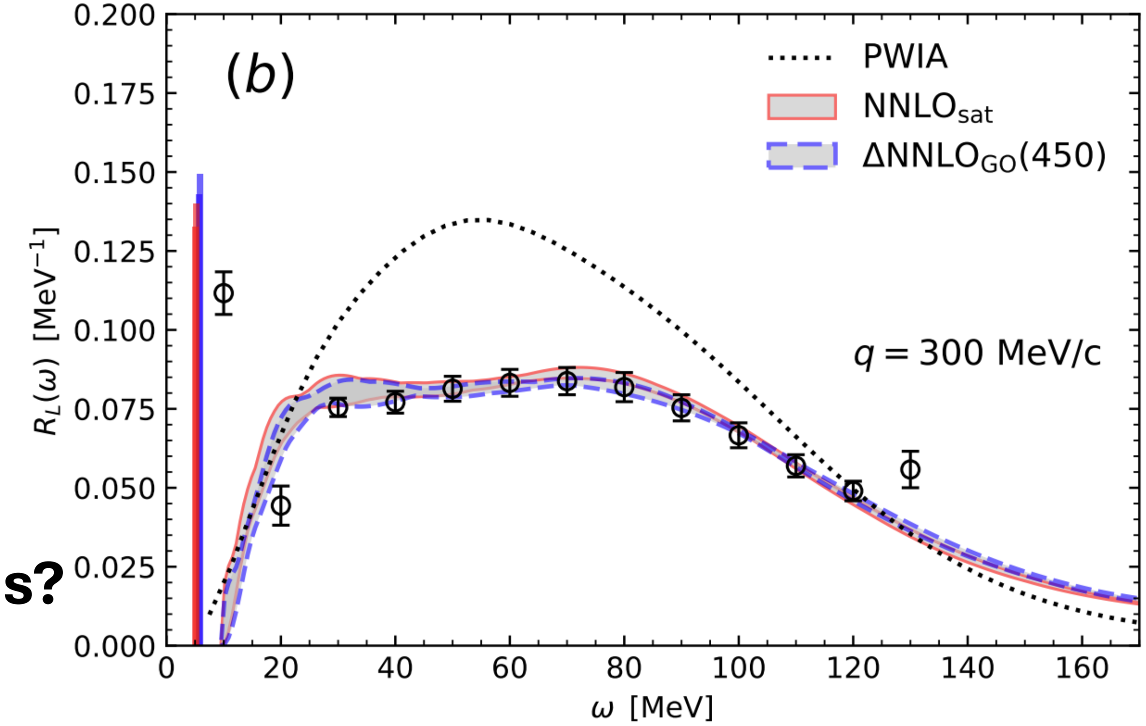
Inversion

Quasi-elastic $e^- - {}^{40}\mathrm{Ca}$ scattering

- Invert integral transform imposing smoothness to obtain response

→Works for responses with one simple structure

→Mathematically ill-posed: **Uncertainties?**



Sobczyk et al., Phys.Rev.Lett. 127 (2021) 7, 072501

Expansion of the integral transform

$$K(\omega, \sigma) = \sum_{k}^{\cancel{\infty}\ \mathrm{N}} c_k(\sigma) T_k(\omega)$$

$$\Phi(\sigma) = \int d\omega\, K(\omega, \sigma) R(\omega) = \sum_{k}^{\cancel{\infty}\ \mathrm{N}} c_k(\sigma) m_k$$

$$m_k = \int d\omega\, T_k(\omega) R(\omega) = \langle \Psi_0 | O^\dagger T_k(H) O | \Psi_0 \rangle$$

The Chebyshev approach

- Suppose you bin the response $$h(\eta, \Delta) = \int d\omega R(\mathbf{q}, \omega) f(\omega, \eta; \Delta)$$

- And the integral transform expanded in Chebyshev polynomials

$$\tilde{h}^{\Lambda}(\eta; \Delta) = \int \int d\sigma d\omega \, K(\omega, \sigma; \lambda) R(\mathbf{q}, \omega) f(\omega, \eta; \Delta)$$

- Then you obtain mathematically sound lower and upper **error limits**

$$\tilde{h}^{\Lambda}(\eta; \Delta - \Lambda) - Q^0 \Sigma \leq h(\eta, \Delta) \leq \tilde{h}^{\Lambda}(\eta; \Delta + \Lambda) + Q^0 \Sigma$$

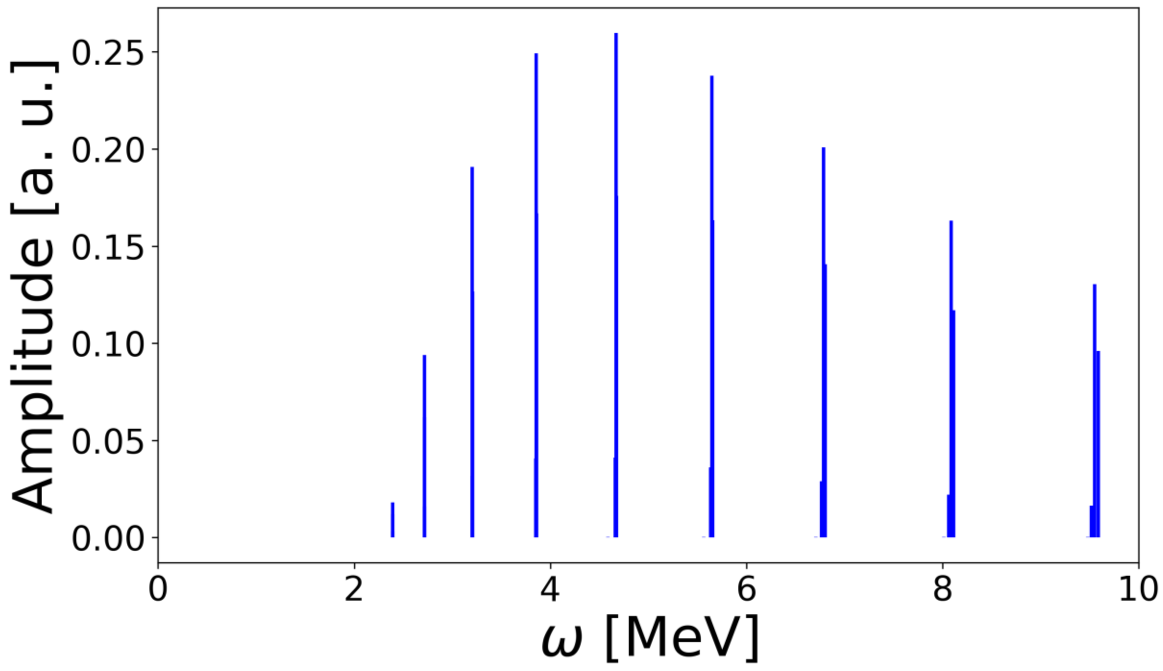
Sobczyk and Roggero, Phys. Rev. E **105** (2022) 5, 055310

Binning Strategy

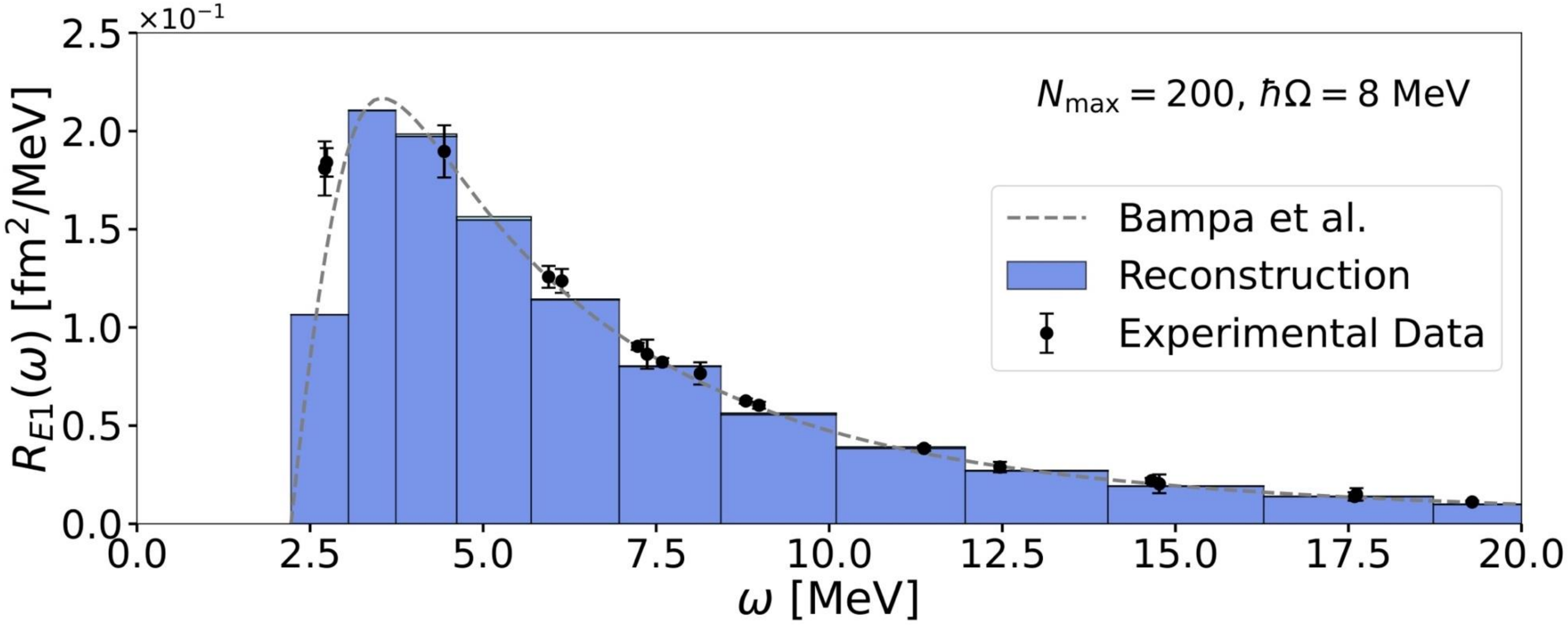
- Each bin should contain a similar number of eigenvalues (>0)
- Bin edges should be in between eigenvalue clusters to minimize error

→Need access to the eigenvalue density

→Regularized density of states can be calculated in the same framework

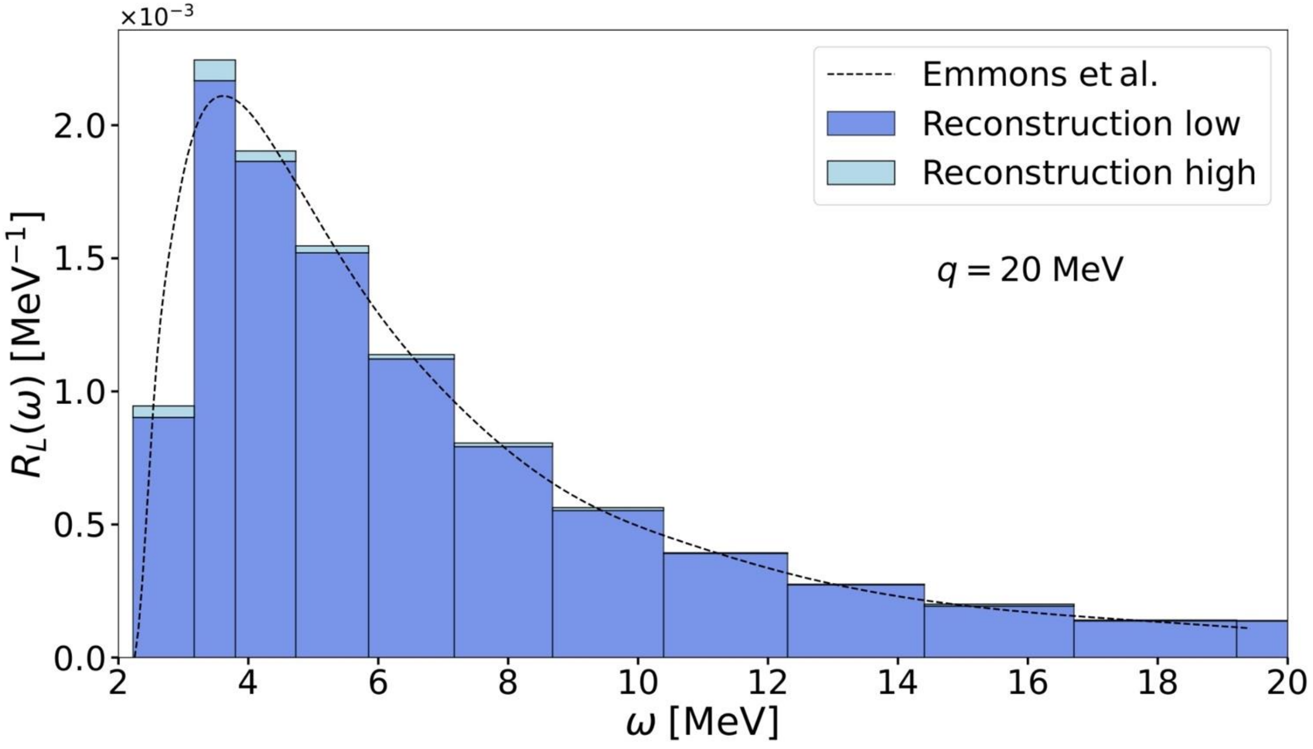


Deuteron E1



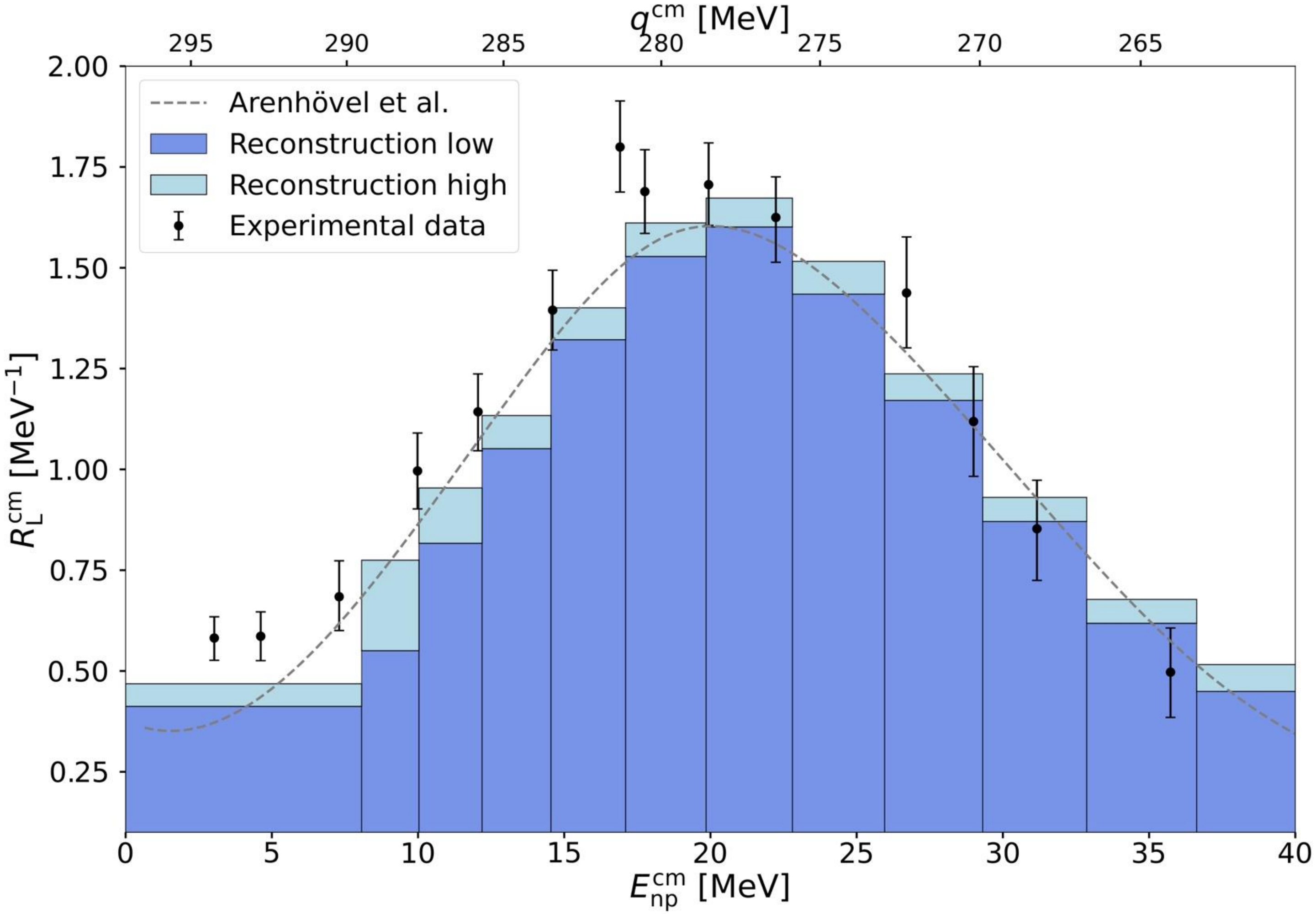
Few Body Syst. Suppl. 3, 1 (1991); PhysRevC.84.034005 (2001)

Deuteron longitudinal response



Emmons et al., J. Phys. G 48, 035101 (2021)

Deuteron longitudinal response



B. P. Quinn et al., Phys. Rev. C 37, 1609 (1988)

Summary

- Chebyshev expansion produces responses with errors
- Errors can be controlled by choosing a sensible binning
- Works well in the deuteron (test case)
- Extend to relevant nuclei ($^{16}\mathrm{O}$, $^{40}\mathrm{Ar}$, …)
- Extend to complicated, narrow responses inaccessible to regular integral transform approaches

Density of states estimation

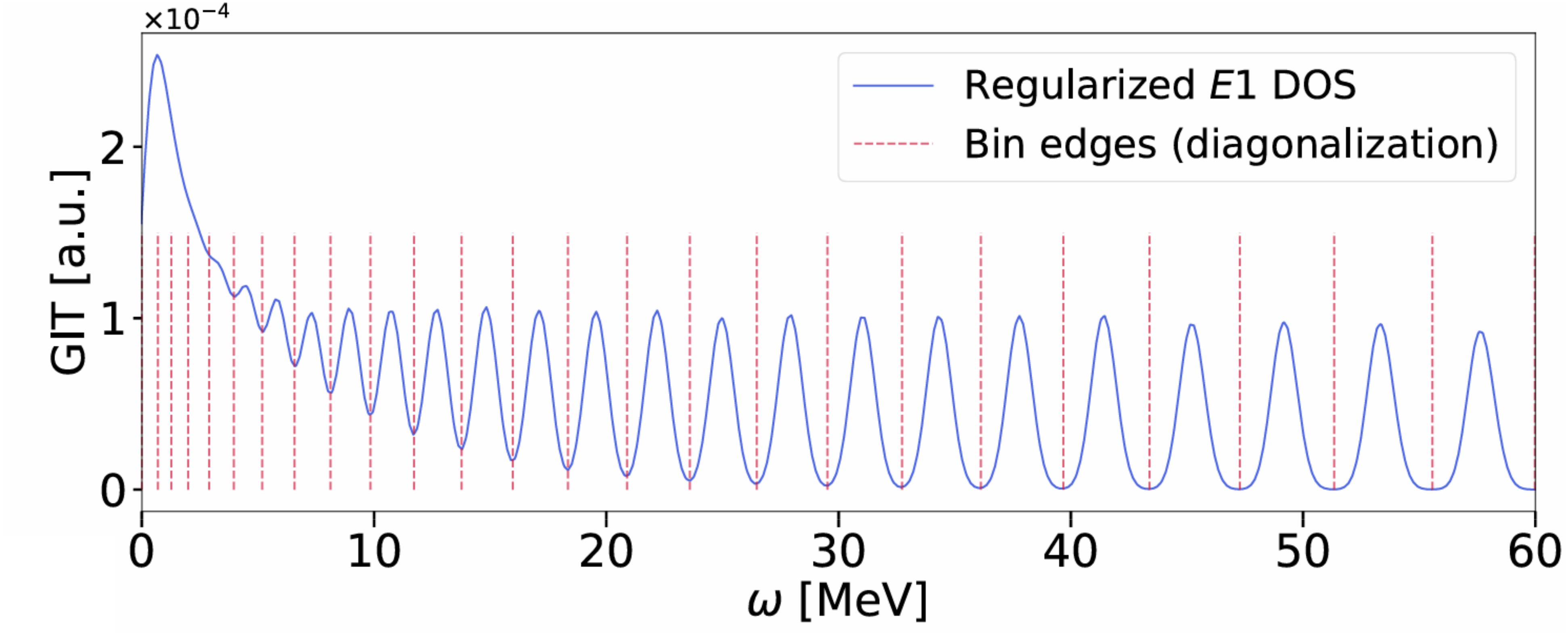
We compute moments of the form $\langle\Psi_0|\, O^\dagger H^k O \,|\Psi_0\rangle$

$$O\,|\Psi_0\rangle = \sum_n d_n\,|\Psi_n\rangle \qquad\qquad H^k O\,|\Psi_0\rangle = \sum_n d_n E_n^k\,|\Psi_n\rangle$$

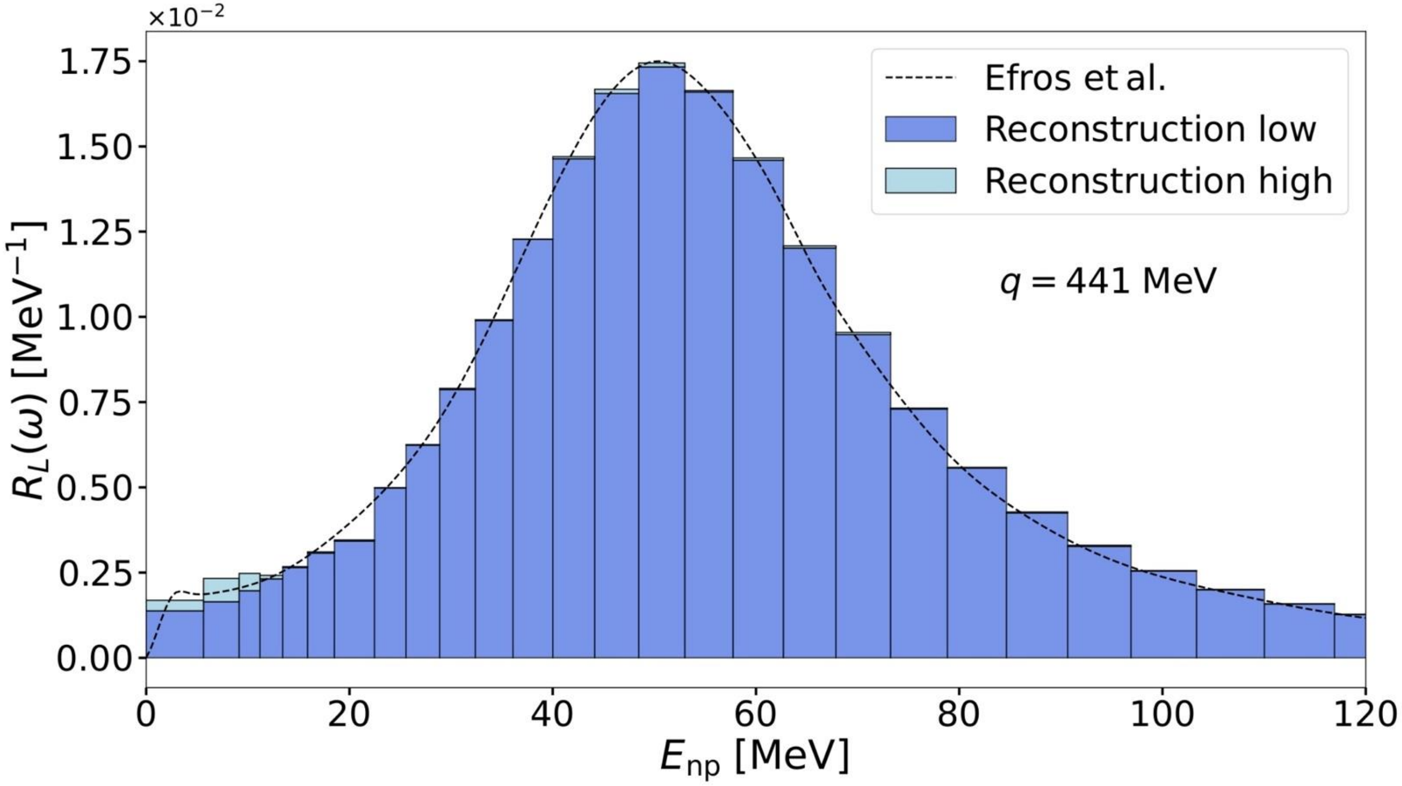
Draw $O\,|\Psi_0\rangle$ randomly so each eigenstate contributes equally to the moments

→Regularized estimate for the density of states

Regularized density of states in the deuteron



Deuteron longitudinal response



Efros et al., Few Body Syst. 14, 151 (1993)